Money and Inflation - The Cagan Model

Econ 208

Lecture 11

March 6, 2007

The Cagan Model

- Cagan's question: Are inflations self-generating or are they caused by monetary expansion?
- The model ($m = \log M$, $p = \log P$, $\mu = \dot{m}$, $\pi =$ expected $\dot{p}$)

$$
\begin{aligned}
m - p &= -\beta\pi && \text{Money supply equals money demand} \\
\mu - \dot{p} &= -\beta\dot{\pi} && \text{Time derivative of first equation} \\
\dot{\pi} &= \alpha \cdot (\dot{p} - \pi) && \text{Adaptive expectations}
\end{aligned}
$$

- Could be generated by the steady state of a Sidrauski model in the additively separable case, taking logs:

$$
\begin{aligned}
M/P &= L(c, r + \pi) \\
m - p &= \log L(c, r + \pi) \\
m - p &= f(\pi) \approx -\beta\pi
\end{aligned}
$$

- Adaptive expectations equivalent to using weighted average of past inflation rates:

$$\pi = \alpha \int_{-\infty}^{t} e^{-\alpha \cdot (t - \tau)} \dot{p}(\tau)\, d\tau$$

The Cagan Model (cont'd)

$$
\begin{aligned}
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\mu - \dot{p} &= -\beta\dot{\pi} && \text{Time derivative of first equation} \\
\dot{\pi} &= \alpha \cdot (\dot{p} - \pi) && \text{Adaptive expectations}
\end{aligned}
$$

- The model takes as given an arbitrary (almost everywhere differentiable) path of $m(t)$ and an initial expectation π_0. Need to solve for the equilibrium time paths of $p(t)$ and $\pi(t)$.
- Example: Constant money growth: $\mu(t) = \mu$, constant. Then there is a steady state equilibrium in which

$$p(t) = m(t) + \beta\mu \text{ and } \pi(t) = \mu$$

- But is this steady state stable? That is, if we start with $\pi_0 \neq \mu$, does $\pi(t)$ converge to μ?
- The answer depends on the values of α and β.

The Cagan Model (cont'd)

$$
\begin{aligned}
m - p &= -\beta\pi && \text{Money supply equals money demand} \\
\mu - \dot{p} &= -\beta\dot{\pi} && \text{Time derivative of first equation} \\
\dot{\pi} &= \alpha \cdot (\dot{p} - \pi) && \text{Adaptive expectations}
\end{aligned}
$$

- From MS = MD and Adaptive expectations:

$$\dot{\pi} = \alpha\left(\mu + \beta\dot{\pi} - \pi\right), \text{ which can be written as:}$$

$$\dot{\pi} = \frac{\alpha}{1 - \alpha\beta}\left(\mu - \pi\right)$$

- This is a linear differential equation, which has a unique solution given any initial value π_0 and a unique rest point $\pi = \mu$.
- If $\alpha\beta < 1$ the rest point is stable, so inflation is a monetary phenomenon.
- If $\alpha\beta > 1$ it is unstable, and π explodes *independently of monetary policy*, as does $\dot{p}$, so inflation is an expectational phenomenon.
- Cagan estimated $\alpha\beta < 1$ for the interwar hyperinflation cases he studied.

Seigniorage

- The implicit tax revenue accruing to the issuer of money, equal to $\dot{M}/P$
- Suppose we take as exogenous not the rate of monetary expansion but the required seigniorage δ
- This is more realistic in the hyperinflation cases, where fiscal pressures were clearly driving monetary expansion

$$
\begin{aligned}
\dot{M}/P &= \delta, \text{ which can be expressed as} \\
\frac{\dot{M}}{M}.\frac{M}{P} &= \delta, \text{ which together with MS=MD yields} \\
\mu e^{-\beta\pi} &= \delta, \text{ or:} \\
\mu &= \delta e^{\beta\pi}, \text{ and, since we still have } \dot{\pi} = \frac{\alpha}{1-\alpha\beta}(\mu - \pi) \\
\dot{\pi} &= \frac{\alpha}{1-\alpha\beta}\left(\delta e^{\beta\pi} - \pi\right)
\end{aligned}
$$

which has **two** rest points, if any!

Seigniorage (cont'd)

- Interpretation of multiple equilibrium: When expected inflation is high, the demand for money is low, so tax base for seigniorage is low, so the tax rate (actual inflation) must be high.
- Which equilibrium is stable? H or L?
- Again, this depends on the product $\alpha\beta$

(a) $\alpha\beta < 1$
L is locally stable, H is unstable
Given $\pi_0 < \pi^H$, $\pi(t) \to \pi^L$
Given $\pi_0 > \pi^H$, $\pi(t) \to \infty$

(b) $\alpha\beta > 1$
H is locally stable, L is unstable
Given $\pi_0 > \pi^L$, $\pi(t) \to \pi^H$
Given $\pi_0 < \pi^L$, $\pi(t) \to -\infty$

Comparative statics with seigniorage

- Assume the "stable" case (a): $\alpha\beta < 1$
- When $\delta \uparrow$ then $\pi^L \uparrow$ (μ jumps, followed by a further increase in μ and π)
- What happens to $\dot{p} = \mu + \beta\dot{\pi}$?
- In the "unstable case (b): $\alpha\beta > 1$ you get wierd comparative statics
- Maximal seigniorage: when the two curves are tangent
  - in this case you get one-sided stability
  - this is a fragile equilibrium

Cagan with Perfect Foresight

The case of exogenous monetary expansion

Now the model is:

$$
\begin{aligned}
m - p &= -\beta\pi \qquad &&\text{Money supply equals money demand} \\
\pi &= \dot{p} &&\text{Perfect foresight}
\end{aligned}
$$

$$
\begin{aligned}
m - p &= -\beta\dot{p}, \text{ or} \\
\dot{p} &= (1/\beta)(p - m)
\end{aligned}
$$

- Given a time path for $m(t)$ this differential equation has a unique solution for **any** given initial value p_0.
- This means the time path of the price level is *indeterminate*.
- Almost all equilibria are unstable
- So perfect foresight seems to **compound** the possible instability problem.

Cagan with Perfect Foresight

The case of exogenous monetary expansion (cont'd)

$$\dot{p} = (1/\beta)(p - m)$$

- Example: constant money supply: $m(t) = m_0$ constant
  In this case the equilibrium is

  $$p(t) = (p_0 - m_0)e^{(1/\beta)t} + m_0$$

  which yields the stable path $p(t) = m_0$ if the initial price level is $p_0 = m_0$ but yields an unstable path for all other initial price levels.
- More generally, for any path of $m(t)$ and any p_0 there is one stable equilibrium, known as the "forward equilibrium":

  $$p^f(t) = (1/\beta)\int_t^\infty e^{(1/\beta)(t-s)}m(s)\,ds$$

  and a continuum of "backward" equilibria:

  $$p^b(t) = p_0 e^{(1/\beta)t} - (1/\beta)\int_0^t e^{(1/\beta)(t-s)}m(s)\,ds,$$

  one for each p_0
- This raises the question of what to do when the equilibrium is indeterminate?