Aggregate Supply and Demand with Rational Expectations

Econ 208

Lecture 17

April 5, 2007
The expectations-augmented Phillips Curve

We can add a price-shock $\varepsilon_t^s$ (0-mean iid) to the (one-period) Fischer model:

$$p_t = E_{t-1}p_t + \phi E_{t-1} (y_t - y_t^*) + \varepsilon_t^s$$

Subtract $p_{t-1}$ from each side:

$$p_t - p_{t-1} = E_{t-1} (p_t - p_{t-1}) + \phi E_{t-1} (y_t - y_t^*) + \varepsilon_t^s,$$

or

$$\pi_t = E_{t-1} \pi_t + \phi E_{t-1} (y_t - y_t^*) + \varepsilon_t^s$$

(EAPC)

**Policy Ineffectiveness:** *Demand-management policy cannot affect the expected value of real output $E_{t-1}y_t$.*

This proposition follows from just two assumptions: EAPC and rational expectations. Proof: Take expectations of both sides of the EAPC, conditional on $t-1$ information:

$$E_{t-1} \pi_t = E_{t-1} \pi_t + \phi E_{t-1} (y_t - y_t^*) + E_{t-1} \varepsilon_t^s$$

$$0 = \phi E_{t-1} (y_t - y_t^*)$$

$$E_{t-1}y_t = E_{t-1}y_t^*$$

and $y_t^*$ is capacity output, which is not affected by demand-management policy.
How policy affects aggregate volatility

Constructing a complete rational-expectations model

Although policy cannot affect the expected level of output beyond the period of price contracts, it can affect the variance of output. Suppose aggregate demand is:

\[ y_t = m_t - p_t + \varepsilon^d_t \]  

(AD)

Suppose demand is managed by the central bank in such a way that:

\[ m_t = \alpha m_{t-1} + \beta y_t, \quad 0 < \alpha < 1 \]  

(MP)

where the central bank can only control the feedback parameter \( \beta \). Suppose \( y^*_t = 0 \) always. Then

\[ p_t = E_{t-1} p_t + \phi E_{t-1} y_t + \varepsilon^s_t \]  

(EAPC)

This is a rational-expectations model, which can be solved for the three endogenous variables \((y_t, p_t, m_t)\) each period. Once we have those solutions we see how monetary policy affects:

\[ E_{t-1} y_t, E_{t-1} p_t, E_{t-1} m_t \]  

(conditional expectations)

\[ E y_t, E p_t, E m_t \]  

(unconditional expectations)

\[ \text{var}_{t-1} (y_t), \text{var}_{t-1} (p_t), \text{var}_{t-1} (m_t) \]  

(conditional variances)

\[ \text{var} (y_t), \text{var} (p_t), \text{var} (m_t) \]  

(unconditional variances)
How policy affects aggregate volatility
Solving the rational-expectations model

\[ p_t = E_{t-1}p_t + \phi E_{t-1}y_t + \epsilon_t^s, \quad m_t = \alpha m_{t-1} + \beta y_t, \quad y_t = m_t - p_t + \epsilon_t^d \]

Taking conditional expectations we have:

\[ E_{t-1}y_t = 0, \quad E_{t-1}m_t = \alpha m_{t-1}, \quad E_{t-1}p_t = E_{t-1}m_t \]

Putting these into EAPC we get:

\[ p_t = \alpha m_{t-1} + \epsilon_t^s \] (solution for \( p \))

Putting these into AD we get:

\[ y_t = \alpha m_{t-1} + \beta y_t - \alpha m_{t-1} - \epsilon_t^s + \epsilon_t^d, \text{ or: } y_t = \frac{1}{1-\beta} \left( \epsilon_t^d - \epsilon_t^s \right) \] (solution for \( y \))

Putting these into MP we get:

\[ m_t = \alpha m_{t-1} + \frac{\beta}{1-\beta} \left( \epsilon_t^d - \epsilon_t^s \right) \] (solution for \( m \))
How policy affects aggregate volatility
Calculating the expectations

\[ p_t = \alpha m_{t-1} + \varepsilon_t^s. \quad y_t = \frac{1}{1 - \beta} \left( \varepsilon_t^d - \varepsilon_t^s \right), \quad m_t = \alpha m_{t-1} + \frac{\beta}{1 - \beta} \left( \varepsilon_t^d - \varepsilon_t^s \right) \]

So:

\[ E_{t-1} p_t = \alpha m_{t-1}, \quad E_{t-1} y_t = 0, \quad E_{t-1} m_t = \alpha m_{t-1} \]

Note that:

\[ E_{t-1} m_{t+s} = \alpha E_{t-1} m_{t+s-1}, \quad s = 0, 1, \ldots, \infty \]

where \( E_{t-1} m_{t-1} = m_{t-1} \) is given. The solution to this difference equation is:

\[ E_{t-1} m_{t+s} = \alpha^{s+1} m_{t-1} \]

so the “unconditional expectation” of \( m \) at any date is

\[ Em_t = \lim_{s \to \infty} E_{t-1} m_{t+s} = 0 \]

Likewise

\[ Ey_t = \lim_{s \to \infty} E_{t-1} y_{t+s} = 0 \quad \text{and} \]

\[ Ep_t = \lim_{s \to \infty} E_{t-1} p_{t+s} = \lim_{s \to \infty} E_{t-1} E_{t+s-1} p_{t+s} = \lim_{s \to \infty} E_{t-1} m_{t+s-1} = 0 \]
How policy affects aggregate volatility

The variance of output

\[ y_t = \frac{1}{1-\beta} \left( \varepsilon^d_t - \varepsilon^s_t \right) \]
\[ p_t = \alpha m_{t-1} + \varepsilon^s_t \]
\[ m_t = \alpha m_{t-1} + \frac{\beta}{1-\beta} \left( \varepsilon^d_t - \varepsilon^s_t \right) \]

\[ E_{t-1}y_t = 0 \quad Ey_t = 0 \]
\[ E_{t-1}p_t = \alpha m_{t-1} \quad Ep_t = 0 \]
\[ E_{t-1}m_t = \alpha m_{t-1} \quad Em_t = 0 \]

So

\[ \text{var}_{t-1} (y_t) = E_{t-1} (y_t - E_{t-1}y_t)^2 = E_{t-1} (y_t)^2 = \left( \frac{1}{1-\beta} \right)^2 \left( \sigma^2_d + \sigma^2_s \right) \]
\[ \text{var} (y_t) = E (y_t - Ey_t)^2 = E (y_t)^2 = \left( \frac{1}{1-\beta} \right)^2 \left( \sigma^2_d + \sigma^2_s \right) \]

Therefore policy can affect the variance of output. In particular, \( \text{var}_{t-1} (y_t) \) and \( \text{var} (y_t) \) are both decreasing functions of the feedback parameter \( \beta \), and both are minimized by setting \( \beta = -\infty \). However, there is a tradeoff, because having a super active policy will destabilize the price level.
How policy affects aggregate volatility

The variance of money and the price level

\[ p_t = \alpha m_{t-1} + \varepsilon^s_t \]

\[ m_t = \alpha m_{t-1} + \frac{\beta}{1-\beta} (\varepsilon^d_t - \varepsilon^s_t) \]

so:

\[ \text{var}_{t-1}(m_t) = E_t-1(m_t - \alpha m_{t-1})^2 = \left(\frac{\beta}{1-\beta}\right)^2 (\sigma^2_d + \sigma^2_s) \]

\[ \text{var}(m_t) = \alpha^2 \text{var}(m_{t-1}) + \left(\frac{\beta}{1-\beta}\right)^2 (\sigma^2_d + \sigma^2_s) \]

\[ \text{var}(m_t) = \frac{1}{1-\alpha^2} \left(\frac{\beta}{1-\beta}\right)^2 (\sigma^2_d + \sigma^2_s) \]

and

\[ \text{var}_{t-1}(p_t) = E(p_t - \alpha m_{t-1})^2 = \sigma^2_s \]

\[ \text{var}(p_t) = \alpha^2 \text{var}(m_{t-1}) + \sigma^2_s \]

\[ \text{var}(p_t) = \frac{\alpha^2}{1-\alpha^2} \left(\frac{\beta}{1-\beta}\right)^2 (\sigma^2_d + \sigma^2_s) + \sigma^2_s \]
The Variance Tradeoff

\[ \text{var} (y_t) = \left( \frac{1}{1 - \beta} \right)^2 \left( \sigma_d^2 + \sigma_s^2 \right) \]

\[ \text{var} (p_t) = \frac{\alpha^2}{1 - \alpha^2} \left( \frac{\beta}{1 - \beta} \right)^2 \left( \sigma_d^2 + \sigma_s^2 \right) + \sigma_s^2 \]

reducing \( \beta \) always reduces the variance of output but does not always reduce the variance of the price level. The terms of the tradeoff depend on the relative size of demand and supply shocks:

\[ \frac{\text{var} (p_t)}{\text{var} (y_t)} = \frac{\alpha^2 \beta^2}{1 - \alpha^2} + (1 - \beta)^2 \frac{\sigma_s^2}{(\sigma_s^2 + \sigma_d^2)} \]