1 Introduction

In this chapter, we show how the Schumpeterian paradigm can be used to model the notion of distance to the technological frontier and to develop a theory of cross-country convergence that departs from the neoclassical one. Here, convergence results from the existence of knowledge spillovers from technologically more advanced countries to less advanced countries, and not from decreasing returns in capital accumulation. As we shall see below, this alternative theory of convergence can account for club convergence and twin-peaks: club convergence refers to the fact that only a subset of countries appear to converge towards the technological frontier in terms of their per capita GDP; twin-peaks refer to the fact that the world income distribution becomes increasingly polarized over time. In other words, the framework can explain both convergence and divergence.

The history of cross-country income differences exhibits mixed patterns of convergence and divergence. The most striking pattern over the long run is the “great divergence” - the dramatic widening of the distribution that has taken place since the early 19th Century. Pritchett (1997) estimates that the proportional gap in living standards between the richest and poorest countries grew more than five-fold from 1870 to 1990, and according to the tables in Maddison (2001) the proportional gap between the richest group of countries and the poorest\(^1\) grew from 3 in 1820 to 19 in 1998. But over the second half of the twentieth century this widening seems to have stopped, at least among a large group of nations. In particular, the results of Barro and Sala-i-Martin (1992), Mankiw, Romer and Weil (1992) and Evans (1996) seem to imply that most countries are converging to parallel growth paths.

However, the recent pattern of convergence is not universal. In particular, the gap between the leading countries as a whole and the very poorest countries as a whole has contin-

\(^1\)The richest group was Western Europe in 1820 and the “Euopean Offshoots” (Australia, Canada, New Zealand and the United States) in 1998. The poorest group was Africa in both years.
ued to widen. The proportional gap in per-capita income between Mayer-Foulkes’s (2002) richest and poorest convergence groups grew by a factor of 2.6 between 1960 and 1995, and the proportional gap between Maddison’s richest and poorest groups grew by a factor of 1.75 between 1950 and 1998. Thus as various authors have observed, the history of income differences since the mid 20th Century has been one of “club-convergence”; that is, all rich and most middle-income countries seem to belong to one group, or “convergence club”, with the same long-run growth rate, whereas all other countries seem to have diverse long-run growth rates, all strictly less than that of the convergence club.

The explanation we develop in this chapter for club convergence follows Howitt (2000), who took a cross-sectoral-spillovers variant of the closed-economy Schumpeterian model and allowed the spillovers to cross international as well as intersectoral borders. This international spillover, or “technology transfer”, allows a backward sector in one country to catch up with the current technological frontier whenever it innovates. Because of technology transfer, the further behind the frontier a country is initially, the bigger the average size of its innovations, and therefore the higher its growth rate for a given frequency of innovations. As long as the country continues to innovate at some positive rate, no matter how small, it will eventually grow at the same rate as the leading countries. (Otherwise the gap would continue to rise and therefore the country’s growth rate would continue to rise.) However, countries with poor macroeconomic conditions, legal environment, education system or credit markets will not innovate in equilibrium and therefore they will not benefit from technology transfer, but will instead stagnate. The one source of divergence we shall emphasize in the last section of the chapter are credit constraints which limit the ability of poorer countries to invest in innovations that would allow them to catch-up with the technological frontier.

Before we develop the model we need to address the question of how our framework, in which growth depends on research and development, can be applied to the poorest countries of the world, in which, according to OECD statistics, almost no formal R&D takes place. The key to our answer is that because technological knowledge is often tacit and circumstantially specific, foreign technologies cannot simply be copied and transplanted to another country no cost. Instead, technology transfer requires the receiving country to invest resources in order to master foreign technologies and adapt them to the local environment. Although these investments may not fit the conventional definition of R&D, they play the same role as R&D in an innovation-based growth model; that is, they use resources, including skilled labor with valuable alternative uses, they generate new technological possibilities where they are conducted, and they build on previous knowledge. While it may be the case that

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4 Cohen and Levinthal (1989) and Griffith, Redding and Van Reenen (2001) have also argued that R&D
implementing a foreign technology is somewhat easier than inventing an entirely new one, this is a difference in degree, not in kind. In the interest of simplicity our theory ignores that difference in degree and treats the implementation and adaptation activities undertaken by countries far behind the frontier as being analytically the same as the research and development activities undertaken by countries on or near the technological frontier. For all countries we assign to R&D the role that Nelson and Phelps (1966) assumed was played by human capital, namely that of determining the country’s “absorptive capacity”.5

2 A simple model of knowledge spillovers and convergence

2.1 Production and equilibrium profits

The model is very similar to the one developed in the previous chapter, except for the specification of the innovation technology: we now introduce spillovers from more advanced countries. Time is discrete and individuals live for one period and have linear preferences in income.

There is a final good which is produced by labor and a continuum of specialized intermediate goods according to the production function:

\[
Y_t = L^{1-\alpha} \int_0^1 A_{it}^{1-\alpha} x_{it}^\alpha d\alpha, \quad 0 < \alpha < 1
\]

where \( x_{it} \) is the input of the latest version of intermediate good \( i \), \( A_{it} \) is the productivity parameter associated with it, and where for simplicity we set \( L = 1 \). The final good is used for consumption, as an input to R&D, and also as an input to the production of intermediate goods, and is taken as numeraire.

The final good is produced under perfect competition, so the price of each intermediate good equals its marginal product:

\[
p_{it} = \alpha A_{it}^{1-\alpha} x_{it}^\alpha. \quad (1)
\]

Each intermediate good \( i \) is produced one-for-one by the incumbent innovator (or inter-

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5Grossman and Helpman (1991) and Barro and Sala-i-Martin (1997) also model technology transfer as taking place through a costly investment process, which they portray as imitation; but in these models technology transfer always leads to convergence in growth rates except in special cases studied by Grossman and Helpman where technology transfer is inactive in the long run.
mediate good producer), using the final good as input: thus, one unit of final good in the hands of the incumbent innovator produces one unit of intermediate input. However, the intermediate good producer faces a competitive fringe of firms that can potentially produce one unit of the same intermediate input, with the same quality $A_t(i)$, but using $\chi > 1$ units of final good as input instead of one unit.

Faced with this competitive fringe, the incumbent producer cannot charge more than the limit price:

$$p_{it} = \chi.$$  \hfill (2)

Now putting (1) and (2) together immediately gives the equilibrium intermediate production by firm $i$, namely:

$$x_{it} = \left(\chi/\alpha\right)^{\frac{1}{\alpha-1}} A_{it},$$

which is proportional to the productivity $A_{it}$ of input $i$. This, in turn, determines the equilibrium profit in sector $i$ for given $A_{it}$, namely:

$$\pi_{it} = (p_{it} - 1)x_{it} = \pi A_{it},$$

where $\pi = (\chi - 1)(\chi/\alpha)^{\frac{1}{\alpha-1}}$ is a constant.

### 2.2 Innovation, spillovers and productivity growth

Prior to producing, the intermediate producer in sector $i$ has the opportunity to innovate so as to increase productivity. In fact, for each intermediate sector $i$ at date $t$, there is one person born at the beginning of period $t$, who is capable of producing an innovation in the sector. This person is called the $i^{th}$ innovator, and if she succeeds (innovates) then she will be the $i^{th}$ incumbent in $t$. Let $\mu_{it}$ denote the probability that she succeeds. Then the following innovation technology captures the existence of knowledge spillovers:

$$A_{it} = \begin{cases} \overline{A}_t & \text{with probability } \mu_{it} \\ A_{i,t-1} & \text{with probability } 1 - \mu_{it} \end{cases}$$

where $\overline{A}_t$ is what we call the world technology frontier. This frontier corresponds to an idealized economy in which productivity in all intermediate sectors would always lie at the maximum level

$$\overline{A}_t = \max\{A_{it}\}.$$ 

For simplicity we shall take growth of the frontier productivity to be exogenous and
constant at rate $g$, thus:

$$\overline{A}_t = (1 + g)\overline{A}_{t-1}.$$ 

The fact that a successful innovator gets to implement $\overline{A}_t$ is a manifestation of technology transfer, of the kind that Keller (2002) calls “active”; that is, domestic R&D makes use of ideas developed elsewhere in the world.$^6$

Assume now that the competitive fringe can always produce an intermediate good embodying the previous technology. Then the unsuccessful innovator will earn zero profits, whereas the profit of an incumbent will be $\pi_{it}$.

Let $\overline{A}_t = \int_0^1 A_{it} \, di$ be the average productivity parameter across all intermediate sectors. In equilibrium the probability of innovation will be the same in all intermediate sectors and constant over time: $\mu_{it} = \mu^*$ for all $i$; therefore average productivity will evolve according to:

$$A_t = \mu^* \overline{A}_t + (1 - \mu^*) A_{t-1}. \tag{3}$$

### 2.3 Distance to frontier and convergence

We now introduce an important notion which shall be used repeatedly in the following chapters of the book, that of distance to frontier (or proximity to frontier). Proximity to frontier is defined in each sector $i$ by the ratio of the sector’s productivity to the frontier productivity, namely:

$$a_{it} = \overline{A}_{it}/\overline{A}_t.$$

Then the average proximity to frontier of the domestic economy, simply determined by:

$$a_t = \int_0^1 a_{it} \, di = \overline{A}_t/\overline{A}_t,$$

is an inverse measure of the country’s distance to the technological frontier, or its “technology gap”.

Using the proximity to frontier variable, we can reexpress the productivity dynamics equation as a very simple difference equation, namely:

$$a_t = \mu_t + \frac{(1 - \mu^*)}{1 + g} a_{t-1} \tag{4}$$

$^6$In Aghion-Howitt-Mayer (2005) we explore the more general case:

$$A_{it} = \begin{cases} b\overline{A}_t + (1 - b)A_{t-1} & \text{with probability } \mu_t \\ A_{i,t-1} & \text{with probability } 1 - \mu_t \end{cases},$$

where $b$ is a real number between 0 and 1.
which converges in the long run to the steady-state value:

$$a^* = \frac{(1 + g) \mu^*}{g + \mu^*}$$

(5)

To close the model, we just need to derive the equilibrium innovation probability $\mu^*$.

2.4 Research arbitrage

Suppose that in each sector the R&D investment needed to innovate at any given rate $\mu$ is governed by the cost function:

$$N_t = c(\mu) \overline{A}_t = (\eta \mu + \delta \mu^2 / 2) \overline{A}_t \quad \eta, \delta > 0$$

where $N_t$ is the quantity of final good that must be invested. We multiply $c$ by $\overline{A}_t$ to recognize the “fishing-out” effect; the further ahead the frontier moves the more difficult it is to innovate.

Assume also that:

$$\pi < \eta + \delta$$

This condition guarantees that the equilibrium probability $\mu^*$ will always be less than 1.

Using the fact the equilibrium profit of an innovating firm is equal to

$$\pi \overline{A}_t,$$

the equilibrium innovation rate $\mu^*$ will be the value of $\mu$ that maximizes the expected net payoff:

$$\mu \pi \overline{A}_t - c(\mu) \overline{A}_t.$$  

(6)

There are two cases to consider:

**Case 1:** If:

$$\pi > \eta$$

(7)

then the reward to an innovation is large enough relative to the cost that producers will innovate at a positive rate. That is, the first-order condition for maximizing (6) is:

$$c'(\mu) = \delta$$

whose solution is:

$$\mu^* = (\pi - \eta) / \delta > 0$$
Case 2: If:
\[ \pi \leq \eta \]
then the conditions are so unfavorable to innovation in this country that producers will not innovate. That is, the first-order condition for maximizing (6) has no positive solution, so the maximization problem is solved by setting \( \mu^* = 0 \).

2.5 Convergence, divergence and the equilibrium distance to the frontier

The first result from this model is:

**Result 1:** All countries with \( \pi > \eta \) will grow at the same rate in the long run.

In other words, all countries that innovate at a positive rate will converge to the same growth rate. The intuition underlying this convergence result can be formulated as follows: because of technology transfer, the further behind the frontier a country is initially, the bigger the average size of its innovations, and therefore the higher its growth rate for a given frequency of innovations. As long as the country continues to innovate at some positive rate, no matter how small, it will eventually grow at the same rate as the leading countries. (Otherwise the gap would continue to rise and therefore the country’s growth rate would continue to rise.)

Formally, we get this result because in this case \( \mu^* > 0 \) implies that the country’s steady-state proximity to the frontier (5) will be strictly positive. Since \( a^* = A_t/A_{t-1} \), this means that in the long run the numerator \( A_t \) must grow at the same rate as the denominator \( A_{t-1} \). That is, the country’s productivity \( (A_t) \) will grow at the same rate as the world productivity frontier.

However, we also have:

**Result 2:** All countries with \( \pi \leq \eta \) will stagnate in the long run.

That is, countries with poor macroeconomic conditions, legal environment, education system or credit markets will not innovate in equilibrium and therefore they will not benefit from technology transfer, but will instead stagnate. Formally, for these countries the fact that \( \mu^* = 0 \) means that their equilibrium proximity to the frontier \( a^* \) is zero, which means that their distance to the frontier, which is \( a_t^{-1} \), is rising to infinity.

Together these two results help to explain the facts about club convergence discussed in the introduction above. That is, there is one group of countries that are converging to parallel growth paths (i.e. with identical long-run growth rates), and another group of countries that are falling further and further behind.

Notice that even countries that are converging to parallel growth paths are not necessarily
converging in levels. That is, one country’s steady-state proximity to the frontier (5) can differ from another’s if they have different values of the critical parameters \( \pi, \eta \) and \( \delta \):

**Result 3:** For countries \( \pi \leq \eta \), \( a^* \) is increasing in \( \pi \), decreasing in \( \eta \) and \( \delta \).

Intuitively, if a country for example improves its education system (thereby reducing the cost parameters \( \eta \) and \( \delta \)) it will start to grow faster for a while. As it approaches closer to the frontier the fact that its size of innovations is getting smaller will bring its growth rate back to \( g \), but the end result will be that it is now permanently closer to the frontier. This result helps us to account for that there are systematic and persistent differences across countries in the level of productivity. That is, convergence in levels is not absolute but conditional. In our model, two countries will end up with the same productivity levels in the long run if they have the same parameter values, but not otherwise.

Finally, it follows from (5) that:

**Result 4:** For countries \( \pi \leq \eta \), \( a^* \) is decreasing in \( g \).

That is, a speedup of the global frontier will result in a spreading out of the cross-country productivity distribution. Howitt and Mayer-Foulkes (2005) have used this result to shed light on the “great divergence” discussed in the introduction above. Their argument is that some time around or after the industrial revolution there was a speedup in world technology growth associated with the spread of scientific methodology and its application to industrial R&D. Countries that did not take part directly in this change (those whose parameter values remained the same) eventually benefitted from technology transfer at an increased rate, but only after they fell further behind. In the long run they were able to grow at the new higher rate but only because their increasing distance to the frontier raised the size of innovations in those countries.

Elsewhere (Aghion and Howitt, 2006) we have also used this result to help explain why the gap between Europe and the United States stopped closing some time in the 1970s or 80s and started rising again. Our argument was that from the end of World War II until some time in the 70s or 80s Europe was catching up to the frontier, but that during the 90s there was an acceleration of productivity growth in the United States, associated with the revolution in information technology, which caused the frontier growth rate to increase. Because for the most part this wave of frontier innovations did not initiate in Europe, it could not produce a higher European growth rate until Europe had fallen further behind the frontier.
3 Credit constraints as a source of divergence

The above framework can be further developed by assuming that while the size of innovations increases with the distance to the technological frontier (due to technology transfer), the frequency of innovations depends upon the ratio between the distance to the technological frontier and the current stock of skilled workers. This enriched framework (see Howitt and Mayer-Foulkes, 2002) can explain not only why some countries converge while other countries stagnate but also why different countries may display positive yet divergent growth patterns in the long-run. Benhabib and Spiegel (2002) develop a similar account of divergence and show the importance of human capital in the process.

In this section we instead explore credit constraints as a source of cross-country divergence, based on Aghion, Howitt and Mayer-Foulkes (2004), henceforth AHM. The section first presents a summarized version of the model in AHM, which directly extends the model in the previous section. Then, we discuss empirical results showing the importance of financial development in the convergence process.

Suppose that the world is as portrayed in the previous section, but that research aimed at making an innovation in period $t$ must be done at period $t-1$. If we assume perfectly functioning financial markets then nothing much happens to the model except that the intermediate firms profit must be premultiplied by a discount factor $\beta$ to reflect the fact that the expected returns to R&D occur one period later than the expenditure. But as we already saw in Chapter 6, when credit markets are imperfect, an entrepreneur may face a borrowing constraint that limits her investment to a fixed multiple $\nu$ of her accumulated net wealth. As in Chapter 6, the multiple comes from the possibility that the borrower can, at a cost that is proportional to the size of her investment, decide to defraud her creditors by making arrangements to hide the proceeds of the R&D project in the event of success.

We assume a two-period overlapping-generations structure in which the accumulated net wealth of an entrepreneur is her current wage income $w_t$, and in which there is just one entrepreneur per sector in each country. This means that the further behind the frontier the country falls the less will any entrepreneur be able to invest in R&D relative to what is needed to maintain any given frequency of innovation. What happens in the long run to the country’s growth rate depends upon the interaction between this disadvantage of backwardness, which reduces the frequency of innovations, and the above-described advantage of backwardness, which increases the size of innovations. The lower the cost of defrauding a creditor, that is, the lower the credit multiplier $\nu$, the more likely it is that the disadvantage of backwardness

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7 For simplicity we suppose that everyone has linear intertemporal preferences with a constant discount factor $\beta$. 
will be the dominant force, preventing the country from converging to the frontier growth rate even in the long run. Generally speaking, the greater the degree of financial development of a country the more effective are the institutions and laws that make it difficult to defraud a creditor. Hence the link between financial development and the likelihood that a country will converge to the frontier growth rate.

### 3.1 Theory

More formally, suppose as we already did in Chapter 6 that an entrepreneur with initial wealth \( w_t \) investing in R&D cannot spend more than

\[
\nu w_t
\]

in innovation. This in turn allows her to innovate at most with probability \( \mu \), where:

\[
c(\mu_{t+1}) \overline{A}_{t+1} = \nu w_t.
\]  

(8)

Using the fact that \( w_t \) is proportional to \( A_t \),

\[
w_t = \theta A_t,
\]

dividing (8) through by \( \overline{A}_{t+1} \), and using the fact that \( \overline{A}_t \) grows at rate \( g \), we can reexpress (8) in terms of our proximity to frontier variable, namely:

\[
c(\mu_{t+1}) = \omega a_t,
\]

(9)

where \( \omega = \frac{\nu \theta}{1+g} \). This equation determines \( \mu_{t+1} \) as an increasing function of the proximity to frontier \( a_t \). For example, if we assume the same R&D technology as in the previous section, namely

\[
c(\mu_t) = \eta \mu_t + \delta \mu_t^2 / 2,
\]

we have

\[
\mu_{t+1} = \tilde{\mu}(\omega a_t) = \frac{\sqrt{\eta^2 + 2\delta \omega a_t} - \eta}{\delta}
\]

which is increasing in \( a_t \) and equal to zero for \( \omega = 0 \) or \( a_t = 0 \).

The credit constraint will be binding on R&D investment if \( \tilde{\mu}(\omega a_t) \) is less than the optimal \( \mu^* \) in the absence of credit constraints (here, we are implicitly assuming that \( \mu^* = \)
\[(\pi - \eta)/\delta > 0\]. In that case, the convergence equation becomes:
\[
a_{t+1} = \bar{\mu}(\omega a_t) + \frac{(1 - \bar{\mu}(\omega a_t))}{1 + g} a_t \equiv F_2(a_t)
\]
which is non-linear in \(a_t\). In particular, a country with very low \(a_t\), that is far below the world technology frontier, will converge at a lower rate than a country with higher \(a_t\). And if \(\omega\) is sufficiently small, a credit-constrained country will in fact diverge from the frontier, as
\[
a_{t+1} < a_t
\]
in that case.

3.2 Evidence

AHM test this effect of financial development on convergence by running the following cross-country growth regression:

\[
g_i - g_1 = \beta_0 + \beta_f F_i + \beta_y (y_i - y_1) + \beta_{fy} \cdot F_i \cdot (y_i - y_1) + \beta_x X_i + \varepsilon_i
\]  
(10)

where \(g_i\) denotes the average growth rate of per-capita GDP in country \(i\) over the period 1960 - 1995, \(F_i\) the country’s average level of financial development, \(y_i\) the initial (1960) log of per-capita GDP, \(X_i\) a set of other regressors and \(\varepsilon_i\) a disturbance term with mean zero. Country 1 is the technology leader, which they take to be the United States.

Define \(\hat{y}_i \equiv y_i - y_1\), country \(i\)'s initial relative per-capita GDP. Under the assumption that \(\beta_y + \beta_{fy} F_i \neq 0\) we can rewrite (10) as:

\[
g_i - g_1 = \lambda_i \cdot (\hat{y}_i - \hat{y}_i^*)
\]

where the steady-state value \(\hat{y}_i^*\) is defined by setting the RHS of (10) to zero:

\[
\hat{y}_i^* = -\frac{\beta_0 + \beta_f F_i + \beta_x X_i + \varepsilon_i}{\beta_y + \beta_{fy} F_i}
\]  
(11)

and \(\lambda_i\) is a country-specific convergence parameter:

\[
\lambda_i = \beta_y + \beta_{fy} F_i
\]  
(12)

that depends on financial development.

A country can converge to the frontier growth rate if and only if the growth rate of its
relative per-capita GDP depends negatively on the initial value \( \bar{y}_i \); that is if and only if the convergence parameter \( \lambda_i \) is negative. Thus the likelihood of convergence will increase with financial development, as implied by the above theory, if and only if:

\[
\beta_{fy} < 0. 
\] (13)

The results of running this regression using a sample of 71 countries are shown in Table 1, which indicates that the interaction coefficient \( \beta_{fy} \) is indeed significantly negative for a variety of different measures of financial development and a variety of different conditioning sets \( X \). The estimation is by instrumental variables, using a country’s legal origins, and its legal origins\(^8\) interacted with the initial GDP gap \( (y_i - y_1) \) as instruments for \( F_i \) and \( F_i (y_i - y_1) \). The data, estimation methods and choice of conditioning sets \( X \) are all taken directly from Levine, Loayza and Beck (2000) who found a strongly positive and robust effect of financial intermediation on short-run growth in a regression identical to (10) but without the crucial interaction term \( F_i (y_i - y_1) \) that allows convergence to depend upon the level of financial development.

TABLE 1 HERE

AHM shown that the results of Table 1 are surprisingly robust to different estimation techniques, to discarding outliers, and to including possible interaction effects between the initial GDP gap and other right-hand-side variables.

\footnote{\(^8\)See LaPorta et al. (1998) for a detailed explanation of legal origins and its relevance as an instrument for financial development.}