Thermoelectric transport control with metamaterial composites

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ABSTRACT
The control of thermal and electric currents is important for many devices and applications. Being able to independently direct the two flows under simultaneous thermal and voltage gradients is, however, difficult when coupling via thermoelectric effects is present in the material. Here, we present a general computational scheme for the design of composites whose constituent materials follow a simple circuit theory of in-parallel and in-series connected transport properties capable of enhancing or inhibiting electric and thermal currents in a desired direction. We show that using the geometry of the flow, thermoelectric metamaterials for controlling currents can be designed using individual components. Controlling the thermoelectrically coupled electric and thermal currents in terms of different flow directions can be significantly optimized by exploring the dissimilarity of the transport properties of the different components.

I. INTRODUCTION
Thermal and electric flows are examples of types of transport, which are important for many practical applications. Understanding the fundamental science of electricity and heat and being able to control them in a given system are of utmost importance for improving existing devices and constructing new ones. In many materials, there is a significant Seebeck effect; thus, electric and thermal currents are coupled, giving rise to thermoelectric transport.1–4 Therefore, controlling electric and heat transport independently is challenging.

In order to build the fundamental science of different types of transport for thermoelectric applications, significant efforts have been devoted toward engineering energy and phonon band structures and scattering processes in materials.5–9 This involves extensive first principles simulations in collaboration with experiments for synthesis and characterization. The main goal in the majority of such studies is to understand the microscopic nature of materials and use it to our advantage. In the field of thermoelectricity, for example, the aim is to find materials suitable for devices for power generation and refrigeration with enhanced efficiency of conversion.7–9

Being able to control thermal and electric flows using macroscopic approaches is emerging as an alternative approach, which promises to deliver new types of devices.10,11 Specifically, systems capable of achieving cloaking, rotating, and concentrating of thermal and electric currents have been demonstrated in the laboratory.12–15 Such works have relied on transformation optics techniques, which take advantage of the invariance of fundamental equations, whose solutions are used to design composites for achieving these unique effects.16 Recently, this approach has been extended to thermoelectric transport with proposed metamaterials with similar targeted effects, such as cloaking, rotating, and concentrating of thermoelectric flow.17,18 These efforts are an important step forward in finding new ways to control not only individual electric and thermal flows but also thermoelectrically coupled transport under applied temperature and voltage gradients. Specifically, thermoelectric devices are environmentally friendly and reliable, but their efficiency of energy conversion is rather low due to the interrelated transport properties of the material.1–4 The underlying reason for this situation is the fact that charge carriers carry electricity and heat; thus, it is difficult to separate electric and heat currents. Therefore, finding ways to independently control such currents in thermoelectric materials at a macroscopic scale, beyond electronic structure engineering, gives an alternative in seeking ways to improve the functionality of thermoelectric devices.

The recently proposed thermoelectric metamaterials17,18 can operate regardless of the applied boundary conditions. They can also function in a thermal or electric mode when only temperature of the
voltage gradient is applied. It is important to note that the designed thermoelectric metamaterials affect the currents coupled via the Seebeck effect in the same way. For example, the electric and thermal flows are cloaked in the same manner in the case of a thermoelectric cloak. This is closely related to the specific coordinate transformation being applied to the fundamental thermodynamic equations for the entire transport, as required by transformation optics techniques.

Here, we explore pathways for metamaterials design suitable for separately controlling thermal and electric currents that are thermoelectrically coupled. For this purpose, we utilize knowledge obtained from thermoelectric metamaterials design from transformation optics, although our approach does not rely on the invariance of the transport equations under coordinate transformations. Using materials components with transport properties obeying simple circuit theory for in-parallel and in-series elements and following the geometry of the flow, composite materials for directing thermal and electric flows in different directions can be obtained. This scheme is rather transparent and concerns transport at larger scales. The thermal and electric conductivities are found using straightforward relations for in-parallel and in-series connected elements, which allow promoting or inhibiting the specific type of current in a prescribed direction. A complete separation between the electric and thermal flows, however, is difficult to achieve in practice, since it requires extreme values of the electric and thermal conductivities.

In this paper, we investigate the application of this idea to different scenarios of independently controlling thermoelectrically coupled thermal and electric currents under the application of temperature and voltage gradients. The relationship between the directions of flows, material properties, and degree of current separation are also examined. We show that by adopting the proposed design principles, the transport properties of the constituent materials, such as the electric conductivity, thermal conductivity, and Seebeck coefficient, can be used to improve the independent control of the electric and thermal flows.

II. THEORETICAL MODELING

The main problem considered here involves a material under general thermoelectric boundary conditions as specified in Fig. 1(a). The applied temperature and voltage gradients cause the flow of thermal and electric currents, which in general are coupled via a Seebeck coefficient as described by thermodynamics. This coupling implies that as carriers move along a voltage gradient $V$, heat is transported as well. At the same time, the generated thermal current due to a temperature gradient $V T$ causes the accumulation of carriers on the hot side, which promotes the generation of an electric current. Our goal is to design a composite that will allow separating the two types of flows by sending each current in a different direction, as schematically shown in Fig. 1(b).

To achieve this goal, we consider diffusive thermoelectric transport as described by basic thermodynamics:\textsuperscript{19,20}

$$ J = - \sigma \cdot \nabla \mu - \kappa \cdot S \cdot \nabla T, \quad (1) $$

$$ J_Q = - \kappa \cdot \nabla T + T S^T \cdot J, \quad (2) $$

where $J$ is the electric current density, $J_Q$ is the heat current density, $\sigma$ is the electrical conductivity tensor, $\kappa$ is the thermal conductivity tensor, and $S$ is the Seebeck coefficient tensor ($S^T$ is the transposed tensor). The above constitutive equations follow Onsager’s theory\textsuperscript{19,20} reflecting the linear relations between currents and generalized forces, and they are consistent with Ohm’s law and Fourier’s law. The governing equations for thermoelectric transport phenomena are given, respectively, below:

$$ \nabla \cdot J = 0, \quad \nabla \cdot J_Q = - \nabla \cdot J, \quad (3) $$

where $\mu = \mu_C + eV$ is the electrochemical potential ($\mu_C$ is the chemical potential and $eV$ is the electric potential energy).

The constitutive equations together with the basic physics laws make up the framework for transport description regardless of the materials involved, even in cases with inhomogeneity and anisotropy present. The first term in Eq. (1) shows that materials with a large electric conductivity promote a large electric current when $V \mu$ is applied. Similarly, a large thermal current is obtained in materials with a large thermal conductivity under the application of $VT$. Therefore, in order to enhance or inhibit $J$ or $J_Q$ in a certain direction, the electric and thermal conductivities must be large or small accordingly. The issue is that single crystal materials that have large $\sigma$ typically have large $\kappa$, and vice versa. Also, due to the thermoelectric coupling, it is difficult to overcome this problem when one desires to independently direct electric and thermal currents.

We propose that by constructing a composite whose basic unit is a block of four materials, as shown in Fig. 1(c), an independent...
control of $I$ and $Q$, and separation in flow directions obeying Eqs. (1) and (2) can be achieved to a large degree. The basic unit consists of four homogeneous and isotropic materials with properties $(\sigma_A, \kappa_A, S_A)$, $(\sigma_B, \kappa_B, S_B)$, and $J_0$. To ensure the promotion or inhibition of the currents, we demand that these materials be effectively connected in series and parallel such that the resultant conductivities and thermal conductivities in some directions, denoted in Fig. 1(c) as I and II, obey the following relations:

$$\sigma_I = \frac{\sigma_A + \sigma_B}{2}, \quad \sigma_{II} = \frac{2\sigma_A \sigma_B}{\sigma_A + \sigma_B}, \quad \kappa_I = \frac{2\kappa_A \kappa_B}{\kappa_A + \kappa_B}, \quad \kappa_{II} = \frac{\kappa_A + \kappa_B}{2},$$

$$\sigma_{II} = \frac{\sigma_A S_a + \sigma_B S_b}{2}, \quad \kappa_{II} = \frac{2\sigma_A S_a \sigma_B S_b}{\sigma_A S_a + \sigma_B S_b},$$

$$(4)$$

From the above equations, one finds that the conductivity of the four-material unit is bigger in one direction $\sigma_I > \sigma_{II}$, while the thermal conductivity is bigger in the other direction $\kappa_I < \kappa_{II}$. This indicates that larger electric current will flow in direction I when compared to direction II, while the opposite trend in direction is found for the thermal current. The equations for the Seebeck coefficients show that $S_I$ and $S_{II}$ have a similar behavior as $\sigma_I$ and $\sigma_{II}$, and they are also dependent on $S_A$ and $S_B$ of the constituent materials.

Equation (4) indicates that materials with largely different $\sigma_A$, $\sigma_B$ and $\kappa_A, \kappa_B$ are needed in order to achieve an independent flow control. Furthermore, the materials comprising the basic four-component unit in Fig. 1(c) will have their own Seebeck coefficients, which can also affect the currents flows. While it is desirable for the entire initially generated currents to be directed along a given path, the presence of thermoelectric coupling may cause some current diversion from the prescribed directions in Eq. (4). The Seebeck effect reflects the fact that the electric carriers also carry heat in the transport process, and at the same time, the thermal gradient makes charges accumulate on the hot side; thus, an electric current is generated, which is imbedded in the thermodynamic Eq. (4). Nevertheless, $S_I$ and $S_{II}$ may be used as additional design parameters to optimize the process of control of the electric and thermal currents.

To further understand how the dissimilarity in the electric and thermal conductivities affects the transport and to explore the Seebeck coefficients as additional design parameters, the following is defined:

$$\beta_{\sigma} = \frac{\sigma_A}{\sigma_B}, \quad \beta_{\kappa} = \frac{\kappa_A}{\kappa_B}, \quad \beta_s = \frac{S_A}{S_B}.$$  

$$(5)$$

With these parameters, one can quantify how well the separation of the two types of currents can be achieved. For example, if $\beta_{\sigma} \to \infty$ and $\beta_{\kappa} \to 0$, then $I$ flows entirely in direction I, while $J_0$ flows entirely in direction II even in the presence of thermoelectric coupling through the Seebeck coefficient. This is actually not possible to achieve in practice, since nature does not offer materials with transport properties of such extreme values. Thus, materials with very dissimilar thermal and electric conductivities can be used to achieve an approximate degree of separation and control of the electric and thermal currents, which can also be further optimized by using different $\beta_s$ ratios.

### III. RESULTS AND DISCUSSION

Using the design principle discussed above, we show some examples of how the thermoelectrically coupled electric and thermal currents can be directed in different ways under standard TE boundary conditions (specified in Fig. 2). Constructing a composite of elements as shown in Fig. 2(a) allows the electric current to be promoted to flow from the hot $A_1A_2$ side toward the top right side, denoted as $B_1B_2$ (chosen as $B_1B_2 = \frac{A_1A_2}{2}$). At the same time, the thermal current can be promoted to flow from $A_1A_2$ mostly in the bottom right side, denoted as $B_2B_3$ in Fig. 2(a), under standard thermoelectric boundary conditions (to be quantified in what follows).

The design concept can be understood by realizing that the composite consists of a checker-board like pattern consisting of $N$ elements with four types of isotropic and homogeneous materials with $(\sigma_A, \kappa_A, S_A)$, $(\sigma_B, \kappa_B, S_B)$, $(\sigma_I, \kappa_I, S_I)$, $(\sigma_{II}, \kappa_{II}, S_{II})$ [schematics in Fig. 1(c)] and following the in-parallel and in-series scheme from Eq. (4). Specifically, in Fig. 2(a), there are $n = 7$ components, following a layered-like arrangement outlined by the direction of the electric current, with alternating electric conductivities and Seebeck coefficients $\sigma_A, \sigma_{II}$ and $\kappa_A, \kappa_{II}$ and $n = 7$ components, following a layered-like arrangement outlined by the direction of the heat current, with alternating thermal conductivities $\kappa_A$ and $\kappa_{II}$, giving the composite with $N = 33$ elements [right panel in Fig. 2(a)]. To better understand the operation of the scheme in Eq. (4), the Seebeck coefficients in the examples in Fig. 2 are chosen to be the same everywhere $S_A = S_B = S$.

To demonstrate this type of control current, we perform simulations of the electric and thermal currents as a function of space from Eqs. (1) to (3) while the constitutive relations in Eq. (3) are satisfied by using a finite element method equations solver as implemented in the COMSOL code. For this purpose, the chosen boundary conditions (schematics in Fig. 1) include the left end being held at $T_L = 300$ K with a current density along the $x$-direction $J^m = 1 A/m^2$ and the right end is electrically grounded and held at $T_R = 285$ K. The top and bottom sides are thermally isolated. The overall size of the composite is $10$ cm by $10$ cm. The results from the calculations for the chosen representative values $\beta_{\sigma} = 20$ and $\beta_{\kappa} = 20$ are shown in Figs. 2(b)–2(d), which indicate that most of the electric current (represented by yellow arrows) now flows in the upper left side, while the thermal current flows in the bottom left side (represented by red arrows), as can be seen by the increased density of yellow and red arrows. Since the composite material is not isotropic and homogeneous, the electric and heat currents are not constant throughout space. Consequently, the intensity of a given current in different locations gives a quantitative measure of its direction and degree of separation. Thus, we define the following parameters for the electric and thermal flows along the $x$-axis:

$$\zeta_e(r) = \frac{J(r)\cdot \hat{x}}{J^m|_{x=0}}, \quad \zeta_t(r) = \frac{Q(r)\cdot \hat{x}}{Q^m|_{x=0}},$$

$$(6)$$

where $J(r)$ is the electric current density at some location $r = (x, y)$, $\hat{x}$ is the unit vector along the $x$-direction, and $J^m|_{x=0}$ is
FIG. 2. (a) Schematics of a composite made with \( n = 7 \) layers with homogeneous thermal conductivities with \( \kappa_A \) (dark blue), \( \kappa_B \) (light blue), and \( n = 7 \) layers with homogeneous electric conductivities with \( \sigma_A \), \( S_A \) (vertical lines) and \( \sigma_B \), \( S_B \) (horizontal lines). The resulting composite contains \( N = 33 \) elements with four types of homogeneous properties \( (\sigma_A, \kappa_A, S_A) \), \( (\sigma_A, \kappa_B, S_B) \), \( (\sigma_B, \kappa_A, S_A) \), and \( (\sigma_B, \kappa_B, S_B) \). (b) Results from numerical simulations for the composite from (a) for the electric (yellow arrows) and thermal (red arrows) currents. The background surface plot is the temperature distribution. (c) Density plots of \( \zeta_e \) defined in Eq. (6) for the composite from (b). (d) Density plots of \( \zeta_Q \) defined in Eq. (6) for the composite from (b). (e) Schematics of a composite made of arranging \( n = 7 \) layered-like components with homogeneous thermal conductivities with \( \kappa_A \) (dark blue), \( \kappa_B \) (light blue), and \( n = 7 \) layered-like with homogeneous electric conductivities with \( \sigma_A \), \( S_A \) (vertical lines) and \( \sigma_B \), \( S_B \) (horizontal lines). The term “layered-like” is intended to indicate the direction followed by the currents. The resulting composite contains \( N = 33 \) elements with four types of homogeneous properties \( (\sigma_A, \kappa_A, S_A) \), \( (\sigma_A, \kappa_B, S_B) \), \( (\sigma_B, \kappa_A, S_A) \), and \( (\sigma_B, \kappa_B, S_B) \). (f) Results from numerical simulations for the composite from (e) for the electric (yellow arrows) and thermal (red arrows) currents. The background surface plot is the temperature distribution. (g) Density plots of \( \zeta_e \) for the composite from (f). (h) Density plots of \( \zeta_Q \) for the composite from (f). The values taken in the simulations are \( \beta_e = 20 \), \( \sigma_A = 2000 \, \Omega^{-1} \cdot m^{-1} \), \( \sigma_B = 100 \, \Omega^{-1} \cdot m^{-1} \), \( \kappa_A = 20 \, W \cdot m^{-1} \cdot K^{-1} \), \( \kappa_B = 1 \, W \cdot m^{-1} \cdot K^{-1} \), \( \beta_B = 1 \), and \( S_A = S_B = 10^{-4} \, \Omega \). The boundary conditions for the simulations are \( T_L = 300 \, K \), and \( T_R = 285 \, K \), and electric current flow from the \( T_B \) side is \( J = 1 \, A/m^2 \). The sides \( A_1 A_2 \) and \( B_1 B_2 \) labeling are the same for all panels on the top row.

the magnitude of the incoming electric current density at \( x = 0 \) (fixed by the initial conditions). Thus, \( \zeta_e \) and \( \zeta_Q \) depict the relative strength of electric and heat currents at any given points in the region with respect to the incoming currents at the left boundary. Currents with larger density than the incoming ones correspond to \( \zeta_e, \zeta_Q > 1 \), while currents with lesser density than the incoming ones correspond to \( \zeta_e, \zeta_Q < 1 \). The same notation is implied for the heat current density. In Figs. 2(c) and 2(d), we show the surface plots for \( \zeta_e(r) \) and \( \zeta_Q(r) \), respectively, that correspond to the results displayed in Fig. 2(b). Clearly, the intensities of the electric and heat currents are not uniform. The electric current is guided along the layers with high electric conductivity with intensity increasing toward the \( B_1 B_2 \) side. The heat current is guided along the layers with high thermal conductivity with intensity increasing toward the \( B_1 B_2 \) side. Our subsequent calculations show that due to the thermoelectric boundary condition, \( J(r) \cdot \hat{y} \) and \( J_Q(r) \cdot \hat{y} \) are several orders of magnitude smaller than \( J(r) \cdot \hat{x} \) and \( J_Q(r) \cdot \hat{x} \); thus, they do not contribute to the transport significantly and are not considered further.

In Fig. 2(e), we offer another example of thermoelectric flow control using a different type of metamaterial composite. In the case of \( n = 7 \) layered-like elements with alternating electrical conductivities and Seebeck coefficients \( \sigma_A, S_A \) and \( \sigma_B, S_B \) and \( n = 7 \) layered-like elements with alternating thermal conductivities \( \kappa_A \) and \( \kappa_B \), the resultant system [right panel of Fig. 2(e)] has a diamond-like shape in the center with most of the electric current passing around its top corner and most of the heat current passing around its bottom corner. The results from the simulations for this system under the same boundary conditions as for Fig. 2(c) are shown in Fig. 2(f). The yellow arrows indicate the directional flow of the electric current, while the red arrows indicate the directional flow of the heat current. The diamond-like region in the center has practically no currents inside, and \( J \) passes above that region while \( J_Q \) passes below it, which is further indicated by the intensity parameters in Figs. 2(g) and 2(h).

The directional control of the thermoelectric currents quantified by the parameters \( \zeta_e(r) \) and \( \zeta_Q(r) \) and shown in Fig. 2 depend on the \( \beta_e \) and \( \beta_Q \) ratios of dissimilarity of the transport properties in Eq. (5), as well as the number of elements making up the metamaterial composites. To further examine this behavior, it is more convenient to study how the intensity ratios involving average outgoing current densities behave as a function of \( \beta_e \) and \( \beta_Q \). As a representative example, the directional control in Figs. 2(a)–2(d) is taken,
for which we consider the following parameters for average outgoing currents:  
\[ \zeta_J = \frac{\rho_J}{\rho_J + \rho_B}, \quad \zeta_J = \frac{\rho_J}{\rho_J + \rho_B}, \quad \zeta_J = \frac{\rho_J}{\rho_J + \rho_B}, \quad \zeta_J = \frac{\rho_J}{\rho_J + \rho_B}, \]
where \( \rho_J = \frac{\rho_J}{\rho_J + \rho_B}, \) are the x-components of electric and heat current densities evaluated per unit length for \( B_1B_2 \) and \( B_2B_3 \), respectively.

In Fig. 3, we show how \( \zeta_J \) and \( \zeta_J \) evolve as \( \beta_s \) and \( \beta_s \) are changed for different number of \( n \)-layers. One finds that as \( \beta_s \) increases, \( \zeta_J \) also increases; however, the opposite behavior is found for \( \zeta_J \) as seen from Figs. 3(a) and 3(b). This indicates that the in-series \( \sigma_J \) guides most of \( J \) toward \( B_1B_2 \), while the in-parallel \( \sigma_J \) will inhibit most of \( J \) toward \( B_1B_2 \). The functional dependences for \( \zeta_J \) and \( \zeta_J \) in Figs. 3(c) and 3(d) show an opposite flowing trend for the heat current. Figure 3 also indicates that as \( \beta_s \) and \( \beta_s \) increase, \( \zeta_J \) and \( \zeta_J \) approach 2 asymptotically, while \( \zeta_J \) and \( \zeta_J \) approach 0 asymptotically. Since \( S_1 = S_3 \), the thermoelectric coupling due to the Seebeck effect is the same in all materials; therefore, the separation effects are mainly attributed to \( \sigma \) and \( \kappa \).

This is further understood in terms of flow conservation, such that the electric and heat currents entering and leaving the region must be the same. Given that \( |B_1B_2| = |B_2B_3| = |B_3B_4| \), the outgoing average electric current will have twice the average intensity of the incoming current along \( B_1B_2 \) for sufficiently large \( \beta_s \) and the outgoing average heat current will have twice the average intensity of the incoming heat current along \( B_3B_4 \) for sufficiently large \( \beta_s \). Next, we explore the effects of the Seebeck coefficients for the materials making up the four-material unit for the constructed composite in Fig. 4. The properties of the four materials are now taken when the composite does not have the same Seebeck coefficient everywhere, as discussed above. In Fig. 4, we show how the ratios of average intensities of incoming and outgoing currents behave as a function of \( \beta_s \) and \( \beta_s \).

We note that when \( \beta_s = 1 \), the trend for \( \zeta_J \) and \( \zeta_J \) as a function of \( \beta_s \) is the opposite when compared with the discussed \( \beta_s \neq 1 \) cases, e.g., in Fig. 4(a), the slopes of the \( \zeta_J \) curves are negative for \( \beta_s = 1 \) and positive for \( \beta_s 
eq 1 \). While in Fig. 4(b), the slopes of \( \zeta_J \) curves are positive for \( \beta_s = 1 \) and negative for \( \beta_s 
eq 1 \). Specifically, the electric current is directed toward the \( B_1B_2 \) side, while the electric current at the \( B_2B_3 \) side flows in the negative x-direction (see Fig. 1). Therefore, keeping the electric conductivity the same everywhere is not beneficial since the initially desired directional control for \( J \) (toward the \( B_1B_2 \) side, positive x-axis) is not achieved. Figures 4(c) and 4(d) further shows that the dissimilarity in the Seebeck coefficients for the components of the metamaterial.

FIG. 3. (a) The parameter \( \zeta_J \) as a function of \( \beta_s \). (b) The parameter \( \zeta_J \) as a function of \( \beta_s \). (c) The parameter \( \zeta_J = \frac{C_1}{C_2} \) as a function of \( \beta_s \). The material properties in (a) and (b) are taken as \( \sigma_s = 100 \frac{2}{5}, \beta_s = 20, \kappa_s = 20 \frac{4}{5}, \beta_s = 1, \) and \( S_3 = S_8 = 10^{-4} \frac{2}{5} \). The material properties in (c) and (d) are \( \beta_s = 20, \sigma_s = 2000 \frac{2}{5}, \sigma_s = 100 \frac{2}{5}, \kappa_s = 1 \frac{5}{3} \).

FIG. 4. The parameters (a) \( \zeta_J = \frac{C_1}{C_2} \), (b) \( \zeta_J = \frac{C_1}{C_2} \), (c) \( \zeta_J = \frac{C_1}{C_2} \), and (d) \( \zeta_J = \frac{C_1}{C_2} \) as functions of \( \beta_s \) for several values of \( \beta_s \). Here, \( S_8 = 10^{-4} \frac{2}{5} \). The so-defined parameters correspond to the metamaterial composite from Fig. 2(a). The color legend is the same in all panels.
does not affect significantly the thermal flow density. We find that \( \zeta_{QJ} \) and \( \xi_{QJ} \) are almost constant as a function of \( \beta_0 \). In fact, our calculations indicate that the behavior of \( J_Q \) is primarily determined by the thermal conductivity properties captured in \( \beta_\kappa \). It is interesting to note that for \( \beta_\kappa = 1 \), the thermal flow is isotropic with \( \zeta_{QJ} = \xi_{QJ} = 1 \). This is expected since the thermal conductivity is the same everywhere. However, the larger dissimilarity between \( \kappa_A \) and \( \kappa_B \) promotes almost the entire heat current to flow toward the \( B_3 \) side as evident from the \( \beta_\kappa = 20 \), 40 cases in Figs. 4(c) and 4(d).

For practical realization of such composites, the interface between the different components must be considered. For example, bringing two materials together creates a boundary region, which in turn can interrupt the predesigned flows, and the degree of separation between the thermal and electric currents can become smaller than the predicted one. Therefore, such unwanted effects must be minimized. While the role of the interfaces is dependent on the thickness and it can be introduced in the above developed scheme, experimental and computational studies for metamaterials (such as thermal cloaks, for example) have shown that abrupt interfaces,18,22 which can be realized in the laboratory, have a small effect on the predesigned transport.

IV. CONCLUSIONS

In this work, we have proposed a design scheme to achieve relatively independent control of thermoelectrically coupled electric and thermal currents under standard thermoelectric boundary conditions. Our approach is based on the governing equations from basic thermodynamics, which are suitable for transport at large scales. The directional flow of the currents involves layered-like materials whose geometry controls the directions of the currents. The electric and thermal conductivities of these materials are based on simple circuit theory with in-series and in-parallel connected components that enhance or inhibit the currents in a desired direction. The Seebeck coefficients of the materials can also be used for further optimization of the electric current primarily, since the heat current is mainly determined by the thermal conductivities of the materials. The resultant metamaterial is an inhomogeneous composite with strongly anisotropic properties, which are not readily available in nature but can be achieved from individual homogeneous isotropic materials. The characteristics of the individual materials components are highly dissimilar due to the proposed design. Although the practical realization of such metamaterials may be challenging, the ever-growing materials library, especially in recent years, gives hope that systems with desired properties to achieve targeted directional control of thermoelectric flow may be found. Depending on the requirements for the transport properties of the individual components, one can use metals (high \( \sigma \) and \( \kappa \)), insulators (low \( \sigma \) and \( \kappa \)), as well as different semiconductors, polymers, and alloys, which can answer the various demands for the \( \sigma \) and \( \kappa \) values. The proposed design can be applied to various types of geometries related to the flow of the currents. We note that more studies are necessary in order to see how such composites for directional control of electric and heat currents can be used to enhance the efficiency of thermoelectric conversion. This technique can also be adopted to other coupled phenomena, such as electro-osmosis, thermal-osmosis, and thermophoresis, which further increases the impact of our work.

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DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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