Surface plasmon polaritons in concentric cylindrical structures

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Abstract

Theoretical studies of surface plasmon polaritons in cylindrical structures consisting of concentric layers are presented. The complete set of Maxwell’s equations with appropriate boundary conditions are solved and expressed in a matrix form for a system with a finite number of shells imbedded in an environment. Numerical evaluations of the transcendental equation for the dispersion relation are given to analyze the role of factors such as number of shells, dielectric properties of the materials and the environment, curvature and size of the system.

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1. Introduction

There has been significant interest in understanding the optical properties of nanostructured materials. Experiments have shown that enhanced optical transmission through arrays of nano-holes in metallic films is possible [1]. The optical functionality of nanostructures due to surface plasmon polaritons (SPP) has also been under much consideration. SPP are electromagnetic excitations bound to a metal-dielectric interface and they consist of collective electronic modes (plasmons) and electromagnetic fields [2]. Metals, such as gold and silver, are especially desirable since they can support excitations in the visible light range.

SPP are attractive as applications in biosensors [3], polarization conversions [4], nonlinear optical effects at low light intensities [5], enhanced optical transmission through metallic films [6] and more. In addition, electromagnetic excitations play a fundamental role in describing and understanding van der Waals and Casimir forces between objects because such forces are associated with the zero point energy of the electromagnetic field excitations [7,8]. Much theoretical work has also been focused on considering two-dimensional scattering configurations [9,10] or superlattices of array of cylinders [11,12].

Multilayer quasi-one dimensional structures present new opportunities for SPP functionalities. Technological advances in synthesizing materials such as carbon nanotubes [13], BN nanotubes [14], MoS$_2$ nanotubes [15], and WS$_2$ nanotubes [16] are of particular interests in this regard. Attempts have also been made towards inserting different types of fillings, such as metallic, dielectric or organic materials inside these nanotubes [17–19]. However, much of the progress made so far is related to experimental manipulations and observations of electromagnetic excitations in nanowires [20–22]. Some theoretical reports focused on modeling SPP excitations in cylindrical structures have also been published. These works consider SPP modes in a dielectric cylinder [23,24], in a cylinder with a dielectric core [25], hollow cylinder [26], and a coated cylinder [27].

The purpose of this work is to present theoretical studies of SPP excitations supported by concentric cylindrical layers. We calculate the dispersion relation $\omega(k)$ and analyze the role of size, curvature, and dielectric properties of the different components. Our studies show that there are many possibilities for tailoring the SPP excitations in such systems. These structures allow a wide range of SPP spectra modifications by manipulating the number of layers, the radial dimensions, and/or the dielectric properties. Thus this work can serve as a basis for fundamental understanding of SPP modes in multilayered cylindrical structures. It can also be used to...
interpret, guide, and/or provide additional opportunities for optimal performance in optical applications, such as optical signal amplification and/or electromagnetic mode excitations in scanning near-field optical microscopy [28–31].

The rest of the paper is organized as follows. In Section 2 we describe the model and the general solution for the SPP dispersion relations. In Section 3 numerical calculations are presented to examine the role of size, dielectric properties, thickness, and number of concentric shells of the system. The summary is given in Section 4.

2. Dispersion relations of the electromagnetic modes

The system under consideration consists of \( N \) concentric cylindrical layers, imbedded in an environment medium. The layers have radii \( R_i \) and a finite thickness \( \delta_i \), where \( i = 1, \ldots, N \) — Fig. 1. The dielectric response function for each layer is \( \varepsilon_i(\omega) \), and the dielectric function for the environment is \( \varepsilon_0(\omega) \).

To obtain the electromagnetic modes for the system, we solve the Maxwell’s equations in cylindrical coordinates \((\rho, \theta, z)\) [32] where the axis of the cylinders is along \( z \) — Fig. 1. The finite speed of light \( c \) is also taken into account. The solutions for the electric and magnetic fields along the \( z \)-axis are given as follows:

\[
E_x^j = \sum_{n=0}^{\infty} \left\{ \frac{\alpha_{n,j}}{\varepsilon_0 \varepsilon_j} I_n(\kappa_j \rho) + F_n^j K_n(\kappa_j \rho) \right\} e^{i \omega t} e^{i(kz-\omega t)}
\]

\[
B_z^j = \sum_{n=0}^{\infty} \left\{ \frac{\beta_{n,j}}{\varepsilon_0 \varepsilon_j} I_n(\kappa_j \rho) + G_n^j K_n(\kappa_j \rho) \right\} e^{i \omega t} e^{i(kz-\omega t)}
\]

where \( k \) is the wave vector along the \( z \) axis, \( \varepsilon_j \) and \( \mu_j \) are the dielectric and magnetic functions for each region, \( \omega \) is the frequency of the excitations, and \( \chi_j^2 = k^2 - \varepsilon_j \mu_j \omega^2 \). \( I_n(\kappa_j \rho) \) and \( K_n(\kappa_j \rho) \) are the modified Bessel functions of order \( n \) and \( I_n'(\kappa_j \rho) = \frac{dI_n(\kappa_j \rho)}{d\kappa_j}, \quad K_n'(\kappa_j \rho) = \frac{dK_n(\kappa_j \rho)}{d\kappa_j} \). For region \( j = 1 \) the coefficients \( F_0^j = G_0^j = 0 \) and for region \( j = 2N+1 \) the \( C_{2N+1}^n = D_{2N+1}^n = 0 \) due the requirement of convergency of the Bessel functions at the \( \rho \to 0 \), and \( \rho \to \infty \), respectively. The expressions for \( \alpha_j^j \), \( \beta_j^j \), \( B_j^0 \), and \( B_j^0 \) can be obtained from the Maxwell’s equations.

For each \( n \), there is an eigenmode corresponding to the electromagnetic field supported by the system and characterized with a dispersion relation \( \omega(k) \). Such modes are of collective character, since they describe the combined effect from the collective electron excitations through the dielectric function and light through the Maxwell equations.

Our goal is to find a solution for \( \omega(k) \) in the real non-radiative regime which exists to the right of the light line — \( \omega = ck \). This is done by realizing that the unknown coefficients \( C_j^j, F_j^j, D_j^j, G_j^j \) can be related via the boundary conditions for continuity of \( \varepsilon_j E_j^j, \varepsilon_j E_j^\prime, \varepsilon_j B_j^j/\mu_j \) at each interface. In our further calculations we consider only non-magnetic materials — \( \mu_j = 1 \) for all regions. Thus one obtains a system of equations relating the coefficients between two adjacent regions, such as:

\[
\begin{bmatrix}
C_{n+1}^j & D_{n+1}^j & F_{n+1}^j & G_{n+1}^j
\end{bmatrix} =
\begin{bmatrix}
\alpha_{n+1,j} + \beta_{n+1,j} & \gamma_{n+1,j} + \delta_{n+1,j} & \gamma_{n+1,j} & \delta_{n+1,j}
\end{bmatrix}
\begin{bmatrix}
C_n^j & D_n^j & F_n^j & G_n^j
\end{bmatrix}
\]

where the following substitutions were made:

\[
\begin{align}
\alpha_{n,j} &= \frac{x_{j+1}}{x_j} I_n(\eta_j) K_n(\eta_{j+1}) - I_n(\eta_j) K_n'(\eta_{j+1}) \\
\beta_{n,j} &= \Theta_{j,j+1} I_n(\eta_j) K_n(\eta_{j+1}) \\
\gamma_{n,j} &= \frac{x_{j+1}}{x_j} I_n'(\eta_j) K_n(\eta_{j+1}) - I_n(\eta_j) K_n'(\eta_{j+1}) \\
\delta_{n,j} &= \Theta_{j,j+1} I_n'(\eta_j) K_n(\eta_{j+1}) \\
\lambda_{n,j} &= \Theta_{j,j+1} K_n(\eta_j) K_n(\eta_{j+1}) - K_n(\eta_j) K_n'(\eta_{j+1}) \\
\lambda_{j,n} &= \frac{x_{j+1}}{x_j} K_n'(\eta_j) I_n(\eta_{j+1}) + K_n(\eta_j) I_n'(\eta_{j+1}) \\
\lambda_{j,n} &= \Theta_{j,j+1} K_n(\eta_j) I_n(\eta_{j+1}) \\
\lambda_{n,j} &= \frac{x_{j+1}}{x_j} K_n'(\eta_j) I_n(\eta_{j+1}) + K_n(\eta_j) I_n'(\eta_{j+1}) \\
\lambda_{j,n} &= \Theta_{j,j+1} K_n(\eta_j) I_n(\eta_{j+1})
\end{align}
\]

Also, \( \Theta_{j,j+1} = \frac{m_j^{j+1} + \omega^2 \eta_j}{\kappa_j} \left[ \frac{x_j}{x_{j+1}} - \frac{\eta_j}{x_{j+1}} \right] \).
and \( \phi_{j,j+1} = \frac{\alpha_{j+1}(\eta_{j}^{2} + \beta_{j+1}^{2})}{\rho_{j+1}^{2}} \left[ \frac{1}{\eta_{j}} - \frac{1}{\eta_{j+1}} \right] \). Similar relations can be derived expressing \( C_{n}^{j}, F_{n}^{j}, D_{n}^{j}, G_{n}^{j} \) in terms of \( C_{n}^{j+1}, F_{n}^{j+1}, D_{n}^{j+1}, G_{n}^{j+1} \) simply by interchanging the indices \( j+1, j \rightarrow j, j+1 \) in the coefficients, \( \chi_{j} \leftrightarrow \chi_{j+1} \) and \( \epsilon_{j} \leftrightarrow \epsilon_{j+1} \) in Eqs. (3)–(19). The interface now becomes \( \rho_{j+1} = \rho_{j} + \delta_{j+1} \) and \( \Delta_{j+1} \rightarrow \Delta_{j} = \Delta_{j+1} \rightarrow \Delta_{j} = I'_{n}(\eta_{j})K_{n}(\eta_{j}) - I_{n}(\eta_{j})K'_{n}(\eta_{j}) \).

These expressions allow one to write the following matrix relation:

\[
\begin{pmatrix}
C_{n}^{1} \\
D_{n}^{1} \\
F_{n}^{1} \\
G_{n}^{1}
\end{pmatrix} = 
\begin{pmatrix}
\alpha_{n,C}^{1,2} & \beta_{n,C}^{1,2} & \gamma_{n,C}^{1,2} & \lambda_{n,C}^{1,2} \\
\alpha_{n,D}^{1,2} & \beta_{n,D}^{1,2} & \gamma_{n,D}^{1,2} & \lambda_{n,D}^{1,2} \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix} \times
\begin{pmatrix}
\alpha_{n,F}^{2,3} & \beta_{n,F}^{2,3} & \gamma_{n,F}^{2,3} & \lambda_{n,F}^{2,3} \\
\alpha_{n,G}^{2,3} & \beta_{n,G}^{2,3} & \gamma_{n,G}^{2,3} & \lambda_{n,G}^{2,3} \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix} \times
\begin{pmatrix}
\alpha_{n,C}^{3,2} & \beta_{n,C}^{3,2} & \gamma_{n,C}^{3,2} & \lambda_{n,C}^{3,2} \\
\alpha_{n,D}^{3,2} & \beta_{n,D}^{3,2} & \gamma_{n,D}^{3,2} & \lambda_{n,D}^{3,2} \\
\alpha_{n,F}^{3,2} & \beta_{n,F}^{3,2} & \gamma_{n,F}^{3,2} & \lambda_{n,F}^{3,2} \\
\alpha_{n,G}^{3,2} & \beta_{n,G}^{3,2} & \gamma_{n,G}^{3,2} & \lambda_{n,G}^{3,2}
\end{pmatrix}
\times
\begin{pmatrix}
\omega_{p}^{2} & 0 & 0 & 0 \\
0 & \omega_{p}^{2} & 0 & 0 \\
0 & 0 & \omega_{p}^{2} & 0 \\
0 & 0 & 0 & \omega_{p}^{2}
\end{pmatrix} \times
\begin{pmatrix}
C_{n}^{1} \\
D_{n}^{1} \\
F_{n}^{1} \\
G_{n}^{1}
\end{pmatrix}.
\]

(20)

The zeros in the above matrices correspond to the requirement of convergency of the modified Bessel functions.

A nontrivial solution is obtained by letting the following determinant vanish:

\[
\left| \begin{array}{cc}
\tau_{n} - 1 & \varphi_{n} \\
\varsigma_{n} & \nu_{n} - 1
\end{array} \right| = 0
\]

(21)

where \( \tau_{n}, \varphi_{n}, \varsigma_{n}, \) and \( \nu_{n} \) represent the appropriate elements after the matrix multiplication in Eq. (20) is done. This is actually a transcendental equation which gives the complete solution for the dispersion relation \( \omega(k) \) for the \( N \) layer multilayer concentric cylindrical structure. This formula includes the dielectric characteristics of each cylinder and the environment and it allows the evaluation of the dispersion of any number of cylinders present. In the general case, the models cannot be classified as pure magnetic (TE) or pure electric (TM) ones. Only for the \( n = 0 \) case, for which there is no angular dependence, such separation is possible. This is different from the cases with planar or spherical symmetries where all modes can be characterized to be either TE or TM [8].

3. Numerical evaluations of plasmon polariton dispersions

Analytical solution of Eq. (21) in general is not possible. Here we present numerical evaluations of Eq. (21) in order to determine the influence and importance of the different factors present. First, we investigate how the number of cylindrical layers affects the SPP modes. The dispersion curves for a system with one, two and three layers are calculated and given in Fig. 2. For the calculations, we take that the entire system is imbedded in free space, thus \( \varepsilon_{0}(\omega) = 1 \). The thickness is chosen to be the same for all layers.

For the purpose of the actual calculations in Fig. 2, the following model for the cylindrical layer dielectric function is adopted. We take that each layer is characterized with an improved Drude type dielectric function which also includes interband transitions — \( \varepsilon(\omega) = \varepsilon_{\infty} - \frac{\omega_{p}^{2}}{\omega(\omega + i\Gamma)} - \frac{\Delta\varepsilon\Omega_{L}^{2}}{\omega^{2} - \Delta\varepsilon^{2}\Omega_{L}^{2} + i\omega\Gamma} \) — where \( \varepsilon_{\infty} \) is the dielectric constant for the medium, \( \omega_{p} \) is the plasma frequency and \( \Gamma \) is the electron lifetime. The last term takes into account the interband transitions where \( \Omega_{L}, \Gamma, \) and \( \Delta\varepsilon \) are the frequency, the spectral width of the Lorentz oscillator and its strength, respectively. The parameters \( \Delta\varepsilon, \Omega_{L}, \) and \( \Gamma \) are usually fitted in order to achieve an agreement with experimental data for the dielectric response of the particular structure. Such forms for \( \varepsilon(\omega) \) have been proven to give good results for the dielectric response properties of metallic nanostructures determined experimentally [33,34].
In addition, the damping parameter $\gamma$ is adjusted to take into account the electron scattering from the interface — $\gamma = \gamma_{\text{bulk}} + A v_F / \delta$, where $\gamma_{\text{bulk}}$ is the electron damping in the bulk material and it includes contributions from physical processes such as electron–electron, electron–phonon and electron–impurity interactions. For nanostructured materials, there is an additional contribution from the electron-interface scattering as their size becomes smaller than the electron mean free path. This is usually done by adding the second term in which $v_F$ is the Fermi velocity, $\delta$ is the thickness of the layer, and $A$ is a constant chosen to fit the experimental data [35, 36]. In our studies, we take the available data for all parameters entering $\varepsilon(\omega)$ for silver given in Ref. [34].

The damping parameters $\gamma$ and $\Gamma$ make the dielectric function complex, thus the SPP frequency also becomes complex resulting in a time decay of the electric and magnetic fields. The real part of $\omega$ represents the energy of the SPP modes and the imaginary part of $\omega$, which has a negative sign, represents the damping of the electromagnetic fields. Fig. 2 shows the energy of the SPP excitations supported by the concentric cylindrical structures. The damping of the SPP modes is found to be at least three orders of magnitude smaller than the actual energy and its absolute value increasing with the increase of $k$. Here we only focus on the real part of $\omega(k)$.

For $N = 1$ case, there are two modes for each $n$ — Fig. 2(a). As the number of layers increases, the number of SPP modes for each $n$ also increases as shown on Fig. 2(b) and (c). This is associated with the presence of more interfaces as $N$ increases, resulting in strong interaction between the electromagnetic excitations from the many interfaces and producing the complexity of the spectrum. Each sub-branch contains contributions from the electromagnetic oscillations from all interfaces. Similar results were found in previous work related to thin slabs [37], a cylinder with a dielectric core [25], and coated spherical shells [38].

Fig. 2 also shows that the two sets of modes for one layer are relatively well separated and all SPP modes go to zero as the wave vector $k$ goes to zero. As the number of layers increases, though, many of the SPP excitations become very close to each other, overlap, cross each other or even split indicating the strong interaction between the electromagnetic excitations located at different interfaces. Also, the modes have strong nonlinearity at smaller values of the wave vector. At higher $k$ values they become almost dispersionless. The nonlinear regime extends over larger $k$-region as $N$ is increased.

Another factor that can influence the SPP eigenmodes is the types of material for the cylindrical layers and for the environment. This is included in the model through the characteristic plasmon frequency. We illustrate the role of $\omega_p$ for the layers by taking the simple Drude formula for the dielectric function without the electron damping and without the interband transition term. In this way, only the effect of the plasma frequency on the dispersion spectrum is studied.

Each layer is taken to be made of the same material (thus same $\omega_p$) surrounded by free space — Fig. 3. As $\omega_p$ increases, the density of electrons increases as well. This means that dispersive metals behave more like perfect conductors at larger $\omega_p$. The graph indicates that for higher values of $\omega_p$, there is a larger region before the dispersionless plateau for the modes is achieved. Also for larger plasma frequencies, the SPP energy is almost parallel to the light line for a larger $k$ region. Therefore, for materials with larger plasmon frequency, the SPP excitations will experience the effect of retardation due to the finite $c$ at larger wave vectors $k$ as compared to materials with smaller plasmon frequency.

Next we examine the effects of size and curvature for the $N = 2$ multilayered system. It has been demonstrated that curvature can have profound role on the energy of the different modes supported by structures with symmetries different from planar [25,39–41]. For example, changes in the dispersion energies of the SPP modes have been shown to be possible by varying the size of a cylinder with a dielectric core [25] and the size of prolate spheroidal formations [41]. Enhanced optical transmission in an array of cylindrical rings imbedded in a metal film by decreasing the radius of each ring has also been shown [39,40]. Here we demonstrate that the SPP dispersion changes one can obtain with concentric cylindrical structures can be very versatile. This is traced to the fact that there are more possibilities in terms of combinations of varying thickness and/or radii of different layers that one can modify.

Further we consider the effect of thickness of the layers on the SPP dispersion by keeping the radii the same. We show results for two concentric cylinders for the lowest $n = 0$ — Fig. 4. Similar dependence (but not shown) is found for all higher branches. When the thickness is an order smaller than the radii, the dispersion reaches a plateau at lower values of the wave vector $k$. While for thickness above $1/2$ of the smaller radii, the nonlinearity is much more pronounced until larger $k$-s. In general, as $\delta$ increases the SPP excitations are pushed up at higher energies. Similar studies were done for a cylinder with a dielectric core [25], where the size of the core of one cylinder was varied over a rather narrow range and some change in the dispersion was seen. Here we examine a broader range of thickness values for two cylindrical layers and show that the dispersion can change significantly as larger variations of $\delta$ are allowed.

We also investigate the role of curvature in the system by examining the SPP modes as a function of the radii of the cylinders. In Fig. 5 we present dispersion curves for the lowest $n = 0$ and $n = 1$ modes for two cylindrical layers with different...
radii, taken to increase simultaneously. Both branches are very close to each other and even overlap, as shown earlier (Fig. 2). Fig. 5 indicates that as the overall size of the system increases and the curvature of the system decreases, the surface plasmon polaritons have smaller energy. This behavior is in contrast to the one observed in Fig. 4, where the SPP energy increased as the thickness of the shells increased (overall size increased). Thus there are two competing effects due to curvature and size of concentric shells in determining the dispersion for the modes.

In Fig. 6(a) we show the dispersion curves for the lowest mode $n = 0$ when the outer radius is kept fixed, while the inner one is varied. In Fig. 6(b) the dispersion curves for the same mode (lowest $n = 0$) are shown when the inner radius is kept fixed, while the outer one is varied. Both cases are examples of bringing the two cylindrical layers closer or further apart from each other. It is evident that when the two shells are closer to each other, the SPP energy is decreased and there is a larger region of nonlinearity. At larger wave vectors, the excitations approach the same value regardless of the relative distance between the shells. Fig. 6 also shows that changing the curvature of the system achieved by varying the inner or outer radius can be different. More specifically, when the curvature is increased by $R_2 = \text{const}$ and $R_1$ decreasing, $\omega(k)$ is shifted up in energy. This is in contrast to the case of increasing the curvature by keeping $R_1 = \text{const}$ but decreasing $R_2$ for which $\omega(k)$ is shifted down in energy.

Thus several factors influencing the SPP $\omega(k)$ can be identified in terms of the geometry of the system. These are the thickness of each cylindrical layer, the overall radial size of the structure, the radius of each layer, and the closeness of the two layers with respect to each other. By varying each one of those factors, competing effects in the dispersion can be seen. For example, if the thickness is increased (the layers are brought closer together) $\omega(k)$ is increased, but if $R_1$ or $R_2$ are varied in such a way as to keep the layers closer, $\omega(k)$ is decreased. Also if the overall curvature of the system is decreased by varying $R_1$ and $R_2$ so $R_2 - R_1 = \text{const}$, $\omega(k)$ is pushed down in value, similarly if the curvature is decreased by keeping $R_2 = \text{const}$ and increasing $R_1$, $\omega(k)$ is also lowered in energy, but if the curvature is decreased keeping $R_1 = \text{const}$ and increasing $R_2$, $\omega(k)$ is pushed up in energy.

The fact that the SPP spectrum of multilayered cylinders has many branches for each mode together with the many ways of shifting of the dispersions up or down in energy by varying the geometry of the system can be of great advantage to experimentalists. For example, in scanning near-field optical microscopy a tip is engineered to work at a specific mode [28, 29]. This is usually done by changing the size of the tip or its coating. Our calculations show that for a one-layered cylinder there are only two branches for each mode available — Fig. 1. Thus for each mode range a new tip needs to be designed. The multilayered structures, however, show that many branches become available for each mode as the number of layers is increased and they can be very close to each other. These findings show that the range of modes becomes wider. In addition, if the tip needs to be excited at a specific frequency, different initial polarizations can be used ($n = 0$ or 1, for example) since the branches for the different modes are close in energy to each other or overlap.

4. Conclusions

We have presented calculations for the energy dispersion of the non-radiative surface plasmon polariton modes supported by concentric metallic cylindrical layers. The general expressions obtained here include explicitly the finite speed of light, the number of layers, and the dielectric properties of the materials and the environment. Specific calculations were done for structures for which the limit of a continuous electron gas distribution over every layer is valid and the dielectric response was taken to have the extended Drude form.

The results from this work show that the surface plasmon polariton spectrum becomes much more complex as the number of layers increases. In the multilayered cylindrical system, the energy subbranches for each branch $n$ are a result from strong interaction between electromagnetic oscillations from all interfaces. Many of the modes from different $n$ can cross each other, split into subbranches at a particular wave vector or overlap. Such complexity, particularly the closeness of the branches, might make it difficult to distinguish them experimentally. On the other hand, if a specific mode range is needed, the excitation can be done with different initial polarizations. The numerical evaluations for the $N = 2$ case indicate that varying the dielectric properties, the thickness, and/or the size of one or both shells can change the SPP modes energy significantly. We demonstrate that the SPP energy can be red-shifted or blue-shifted in different ways simply by modifying the size and curvature of the systems.
Fig. 6. Energy dispersion for the lowest \( n = 0 \) SPP eigenmodes: (a) the inner radius is varied; (b) the outer radius is varied. The thickness of each shell is \( \delta = 3 \) nm and the plasmon energy is \( \hbar \omega_p = 14 \) eV. The light line is \( \omega = c k \).

The general expressions we present here can be used for other studies as well. For example, all or some layers can be made to have different thickness and/or different dielectric properties (different \( \omega_p \)). The system can also be submerged into an environment with \( \epsilon_0 \neq 1 \). Thus multilayered cylindrical structures provide many ways of tailoring the optical properties of nanostructures demonstrating their versatility in optical applications.

We also suggest that such cylindrical structures may find applications in scanning near-field optical microscopy as tips. There are many advantages as indicated by our results for doing this — more modes are available for excitations, thus greater range of operation; a range of excitations can be excited by different initial polarizations; blueshifting and redshifting of the dispersion spectra can be achieved in many more ways as compared to using just coated cylindrical tips, thus there are many more possibilities for engineering the system for a desired range of operation.

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