Three-dimensional angle measurement based on propagation vector analysis of digital holography

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We propose what we believe to be a novel method for highly accurate three-dimensional (3D) angle measurement based on propagation vector analysis of digital holography. Three-dimensional rotations in space can be achieved by use of a CCD camera and a multifacet object, which reflects an incident wave into different directions. The propagation vectors of the reflected waves from the object can be extracted by analyzing the object spectrum of the recorded hologram. Any small rotation of the object will induce a change in the propagation vectors in space, which can then be used for 3D angle measurement. Experimental results are presented to verify the idea. © 2007 Optical Society of America

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1. Introduction

Techniques to measure small rotation angles of an object are widely used in both automotive and industrial applications. For example, optical clinometers are used for calibration, alignment, machine setting, boresighting, fire control alignment, and inspection when the tightest tolerances are expected and required [1]. During the past decade, many different angle measurement techniques have been developed. Among them, the most widely used are those based on autocollimators [2,3], interferometers [4–6], and total internal reflection [7–9]. An autocollimator is an optical apparatus that projects an illuminated reticle to infinity and receives the reticle image after reflection from a mirror stuck onto a measured object. Angle information is obtained by comparing the positions between the reflected image of the collimator reticle and the eyepiece reticle. Autocollimators are simple in design; however, the size of the system will be very large if high accuracy is required. Angle-sensitive interferometers are widely used because of their high accuracy and ease of use. Their angle measurement is based on changes in optical path length, which has a nonlinear sinusoidal relationship with the rotation angle; thus sinusoidal error may occur. Furthermore, it is difficult to reduce the size of the system since its resolution is determined by the length of the sine arm. The method based on total internal reflection uses two equal-intensity beams, which are incident upon two separate prisms and reflected in the vicinity of the critical angle. The small rotation angle of the two prisms can be estimated by measuring the reflectance of the beams from the two prisms, but it is quite sensitive to variations in the intensity of the light source or stray light. There are also some other optical techniques based on fringe analysis [10], speckle correlation [11], or orthogonal parallel interference patterns [12].

However, all the above-mentioned techniques were developed and applied for one-dimensional (1D) or two-dimensional (2D) angle measurement. Motivated by the increasing demands on the precision of microsystems, compact three-dimensional (3D) angle measurement systems with high accuracy are needed. However, a simple combination of multiple 1D or 2D angle measurement techniques in one system is usually not the best solution for 3D angle measurement, and it will make the whole system rather complicated in structure. Recently, C. H. Liu et al. [13] proposed a 3D angle measurement method that uses two position-sensitive detectors to measure the spot positions of diffracted waves from a grating. This method is...
based on the measurement of the spot movements on different sensors, and its measurement resolutions around three axes are limited to 0.1–0.2 min within a measurement range of 1–2 degrees.

This paper proposes what we believe to be a novel method for 3D angle measurement based on propagation vector analysis of digital holography, where a single CCD camera is used and high accuracy is achieved. A multifacet object (or more precisely, an object that can reflect or diffract an incident wave to different directions) is particularly used as an indicator of rotation. Any small 3D rotations of the object will induce a change in the propagation vectors in space, which will be manifested by the spatial spectrum of the object. Thus, by evaluating the change of the object spectrum in holograms, change of the wave propagation vectors can be traced and 3D angle measurement is possible. To the best of the authors’ knowledge, this paper presents the first 3D angle measurement method based on a single detector.

2. Principle

Suppose the multifacet object is observed by a digital holographic system based on a Michelson interferometer as shown in Fig. 1. The laser beam is split at beam splitter BS into reference and object beams, and each part is focused by lens L1 onto the focal point F1 or F2. Point F2 is also the front focus of objective L2, so the object is illuminated with a collimated beam. The plane S is imaged to the CCD camera by lens L2. In the reference arm the beam is also collimated by lens L3, which results in a magnified image at the CCD camera of an interference pattern that would exist at S if the object wave were superposed with a plane wave there. Thus an off-axis hologram can be recorded by slightly tilting the reference mirror REF. The object angular spectrum can be easily separated by a bandpass filter if the off-axis angle of the two beams is properly adjusted [14–17], and the object complex amplitude at different plane \( z \), \( E(x, y, z) \), can be reconstructed. From Fourier optics, \( E(x, y, z) \) can be expressed as

\[
E(x, y, z) = \int \int S(\xi, \eta; z) \exp[i2\pi(\xi x + \eta y)] \, d\xi d\eta, \quad (1)
\]

where \( S(\xi, \eta; z) \) is the corresponding angular spectrum of the object wave and \( \xi \) and \( \eta \) are the spatial frequencies of \( x \) and \( y \), respectively. The complex-exponential function \( \exp[i2\pi(\xi x + \eta y)] \) may be regarded as a plane wave propagating with spatial frequencies \( \xi = \cos \alpha / \lambda, \eta = \cos \beta / \lambda, \) and \( \zeta = \cos \gamma / \lambda, \) where \( \cos \alpha, \cos \beta, \) and \( \cos \gamma \) are the direction cosines of the normal propagation vector \( n \) of the plane-wave component, and \( \zeta = (\lambda^2 - \xi^2 - \eta^2) / z \) is the spatial frequency along the \( z \) axis. From above, the field \( E(x, y, z) \) can be viewed as a composition of many plane-wave components propagating with different angles in space and with the complex amplitude of each component equal to \( S(\xi, \eta; z) \). Each point \( (\xi, \eta) \) on \( S(\xi, \eta; z) \) presents a plane-wave component propagating with a unit normal vector

\[
n = [\cos \alpha, \cos \beta, \cos \gamma]^T = \lambda[\xi, \eta, \zeta]^T, \quad (2)
\]

where the superscript \( T \) represents the transpose of a vector.

For simplicity, if a two-facet object is considered, the normal propagation vectors of the two reflected plane-wave components from the object are labeled as \( n_1 \) and \( n_2 \) in an absolute \( x-y-z \) coordinate system. Figure 2(a) shows the top view of the two-facet object in the \( x-y-z \) coordinate system. We also use frame \( A \) or \( x_i, y_i, z_A \) to represent this absolute frame in Figs. 2(b)–2(d). To fulfill 3D angle measurement, another local coordinate system \( x_i, y_i, z_B \) or frame \( B \) is introduced to attach to the object. Thus, as the object is rotating, local frame \( B \) will also change its orientation in space. Thus, by tracing the relative orientation or rotation of frame \( B \) to absolute frame \( A \), a 3D angle can be measured. Note that we can assume that frames \( A \) and \( B \) share the same origin, since only 3D rotations are of interest, and translations will not affect the angle measurement.

It is interesting to note that, except for a constant \( \lambda \) scale difference, the \( x_i-y_i \) projections of the two normal vectors \( n_1 \) and \( n_2 \) are exactly the same as the spatial frequency distribution \( (\xi_i, \eta_i) \) \((i = 1, 2) \) of the object spectrum, as indicated by Fig. 2(b). Thus, by extracting the object spectrum in digital holography and analyzing the direction cosines of the two plane-wave components, the propagation vectors \( n_i = \lambda[\xi_i, \eta_i, \zeta_i]^T \) and \( n_2 = \lambda[\xi_2, \eta_2, \zeta_2]^T \) can be precisely obtained. In practice, the spatial frequencies \((\xi_i, \eta_i)\) can be obtained by either fitting the reconstructed phase information of each object facet to a form \( \phi(x, y) = 2\pi(\xi x + \eta y) \) by a least-squares method or simply evaluating the peak position of the object spectrum.

Note that, although the object is composed of two mirrors, the surface is not a diffuse one; however, the split or the connecting edges between the two mirrors are diffuse, and by observing which we can tell whether the recorded or reconstructed object is in focus or not. By reconstructing the object in focus by the digital holographic method, diffraction of the edges or...
mixing between the two reflected object waves can be avoided. Furthermore, as will be discussed later, it is also possible to use the digital holography method to reconstruct wavefields and analyze propagation vectors of more complex objects at different reconstruction positions, where a straightforward fast Fourier transform (FFT) fringe analysis [10] for the interference pattern (interferogram) would be inadequate.

The obtained phase information represents the phase difference of the two-facet object relative to the reference mirror. Thus it is reasonable to define the $x_A$-$y_A$ plane parallel to the reference mirror and the $z_A$ axis coincident with its normal direction. Note that the phase information $\varphi(x, y)$ of each object facet only gives 2D angle information. In other words, neither digital holography nor the FFT fringe analysis method itself provides 3D angle information. We propose a novel algorithm to achieve 3D angle information by combining the (two) spatial frequency pairs $(\xi_i, \eta_i)$ of the object. Vectors $n_1$ and $n_2$ define a plane $P$ in space, as shown in Fig. 2(c). To evaluate the 3D orientation, another local frame (B) is attached to plane $P$. The three axes of {B} can be specified in frame {A} as

\begin{align*}
{^A}z_B &= (n_1 + n_2)/|n_1 + n_2|, \\
{^A}x_B &= (n_1 - n_2)/|n_1 - n_2|, \\
{^A}y_B &= {^A}z_B \times {^A}x_B, \\
\end{align*}

where the symbol $\times$ means the cross product of two vectors and $| |$ represents the modulus of a vector. Actually, frame {B} can also be defined as any other forms by choosing different local axes, but not limited to Eq. (3). The rotation matrix of frame {B} relative to frame {A} can be calculated by describing the coordinate system {B} in the system {A} as $^B R = [x_B^A, y_B^A, z_B^A]$. Once the rotation matrix is known, we can decompose the object orientation as three rotations taking place about an axis in the fixed reference frame {A}. For example, starting from a position coincident with frame {A}, frame {B} can be obtained by rotating about $z_A$ by an angle $\theta_1$, then rotating about $y_A$ by an angle $\theta_2$, and finally rotating about $x_A$ by an angle $\theta_3$; thus the total transform matrix $^B R$ can be expressed as

\begin{equation}
^B R = R_{z_A}(\theta_1)R_{y_A}(\theta_2)R_{x_A}(\theta_3),
\end{equation}

where $R_i(\theta)$ represents the transfer matrix of a rotation about an axis $i$ by an amount of $\theta$. Expanding the above matrix multiplication and comparing it with the known $3 \times 3$ rotation matrix $^B R = t_{ij}$, $(i, j = 1, 2, 3)$ from Eq. (3), the fixed rotation angles can finally be calculated as

\begin{align*}
\theta_1 &= \arctan(t_{13}, \sqrt{t_{23}^2 + t_{33}^2}), \\
\theta_2 &= \arctan(-t_{23}/\cos \theta_2, t_{33}/\cos \theta_2), \\
\theta_3 &= \arctan(-t_{12}/\cos \theta_2, t_{11}/\cos \theta_2),
\end{align*}

Fig. 2. (Color online) (a) Top view of the object in an absolute $x$-$y$-$z$ frame; (b) propagation vectors of the two-facet object; (c) local frame {B} attached to the object; (d) shift of the propagation vectors due to 3D rotations in space.
where \( \arctan(y, x) \) is a two-argument arctangent function. Thus the 3D orientations of the object can be measured in frame \( \{ A \} \), which is equivalent to an absolute angle measurement. This is referred to as an inverse problem to calculate the rotation angles while the rotation matrix is already known. Relative angle measurement is also possible by tracing the changes of these angles themselves. For example, after a three-dimensional rotation of the object, the two propagation vectors will change to \( \mathbf{n}_1 = \lambda[\xi_1, \eta_1, \zeta_1] \) and \( \mathbf{n}_2 = \lambda[\xi_2, \eta_2, \zeta_2] \), as shown in Fig. 2(d). Similarly as above, the attached local frame \( \{ B \} \) now changes to \( \{ A \} \), whose rotation matrix relative to frame \( \{ A \} \) can be calculated as \( \frac{A}{B} \mathbf{R} \). The change of rotation angles \( \theta_1, \theta_2, \) and \( \theta_3 \) can be easily obtained by following the same angle decomposition process as above and comparing the angles with their initial values. Rotation angles in a different rotation sequence or in any other coordinate systems can also be obtained.

3. Experiments

Our first experiment is aimed at demonstrating that the wave propagation information from a multiple-facet object can be recorded by a single hologram. A USAF-1951 resolution target with an area of 2.62 mm \( \times \) 2.62 mm and 354 \( \times \) 354 pixels is illuminated with a green 532 nm Nd:YAG laser and is positioned 2 mm away from the \( S \) plane as shown in Fig. 1. The resolution target is intentionally split into two parts, which reflect an incident wave into two propagation directions. Figure 3(a) shows the recorded hologram and Fig. 3(b) shows its spectrum. One can clearly see that the two propagation components are separated in spectrum domain and can be easily extracted by use of a filter. The spectrum of the right-down part of the resolution target is filtered and shown in Fig. 3(c), based on which the corresponding amplitude and phase information are reconstructed as Figs. 3(d) and 3(e), respectively. Thus the spatial frequency pair (\( \xi_1, \eta_1 \)) can be obtained by unwrapping and fitting (a portion of) the phase map in Fig. 3(e) with a simple least-squares method [10], and the propagation vectors can be achieved with high precision. Since the 3D angle measurement is based on the correct acquisition of the propagation vectors, it is important to apply a good phase-fitting method. Some other methods [18–20] might also be used for phase fitting here.

Phase-unwrapping errors might occur if phase unwrapping is directly implemented based on the raw phase map, which has dense phase jumps and phase noises. To diminish these possible phase errors, the object spectrum can first be shifted \( N_\xi \) pixels in the \( \xi \) (horizontal) direction and \( M_\eta \) in the \( \eta \) (vertical) direction to near the center of the spectrum, as shown in Fig. 3(f), and the residual fringes on the reconstructed phase map will have lower frequency, as shown in the right-down portion of Fig. 3(g), which can then be unwrapped and fitted to get a residual spatial frequency pair (\( \xi_{1r}, \eta_{1r} \)). The total spatial frequency (\( \xi_1, \eta_1 \)) can be obtained as

\[
\xi_1 = X^{-1}N_\xi + \xi_{1r}, \quad \eta_1 = Y^{-1}M_\eta + \eta_{1r},
\]

where \( X \) (or \( Y \)) is the size of the squared image area and \( N_\xi \) and \( M_\eta \) are the shifted pixel numbers of the spectrum component in the \( \xi \) and \( \eta \) directions, respectively. Similarly, Figs. 3(h) and 3(i) show the reconstruction of the up-left portion of the resolution target. The above clearly show that both the amplitude and phase information of a two-facet object can be obtained by a single hologram. Note that phase information is of more interest here, since it indicates the propagation directions of the reflected waves from the object, although the amplitude information can also be reconstructed in digital holography.

For 3D angle measurement, a similar two-facet object is accomplished by sticking two plane mirrors together. The relative angle \( \alpha \) between the two plane mirrors is about 0.5°. First of all, rotation around the \( x \) axis is studied as shown in Fig. 1. The maximum measurement angle around the \( x \) (or \( y \)) axis of the proposed method is given as

\[
\theta_{\text{max}} = \sin^{-1}\left( \frac{\lambda M}{2 \Delta x} \right),
\]

which is determined by the resolution of the CCD camera \( \Delta x \), the wavelength \( \lambda \), and the magnification.

![Fig. 3](Color online) (a) Hologram of a split resolution target; (b) spectrum of the hologram; (c) filtered spectrum component of the right-down part of the resolution target; (d), (e) reconstructed amplitude and phase images, respectively, from (c); (f) obtained by shifting (c) to near the center of the spectrum domain; (g) reconstructed phase map from (f) containing fringes with lower frequency; (h), (i) reconstructed amplitude and phase images, respectively, of the up-left part of the target.
M of the optical system. In our experiment, if the system is simplified so that $M = 1$, the 7.4 μm resolution of the CCD camera and the 532 nm wavelength determine a 2.06° measurement range around the x (or y) axis.

Figure 4 shows another example to use the proposed method for 3D angle measurement. The hologram (or interferogram) of the two-facet object is shown in Fig. 4(a), and Fig. 4(b) shows its spectrum. In the left part of Fig. 4(b) one can clearly see two separated real spectrum components, which represent the waves propagating in two different directions. Figure 4(c) shows the object spectrum after rotation mostly around the x axis. Figures 4(d)–(f) show the spectrum, phase map, and unwrapped phase map of the right facet, respectively. Thus the spatial frequency pair $(\xi_1, \eta_1)$ can be obtained by fitting the phase map in Fig. 4(f) to a form $\varphi(x, y) = 2\pi[\xi_1 x + \eta_1 y]$ by a least-squares method, and finally the normalized wave propagation vectors $n_1$ can be calculated as $n_1 = [0.015274 \ 0.010582 \ 0.99983]^T$ from Eq. (2). Similarly, Figs. 4(g)–4(i) are the spectrum and the wrapped and unwrapped phase map of the left facet of the object. The normalized wave propagation vectors $n_2$ are calculated as $n_2 = [0.014721 \ 0.0009231 \ 0.99989]^T$. Here we have defined the $x_{A} - y_{A}$ plane parallel to the reference mirror as shown in Fig. 4, and the $z_{A}$ axis is coincident with its normal direction. Following from Eqs. (3)–(5), the absolute angle orientations of frame [B] are calculated as $\theta_1 = 86.724^\circ$, $\theta_2 = 0.8593^\circ$, and $\theta_3 = -0.3296^\circ$. Similarly, the absolute angle orientations from Fig. 4(c) are also calculated as $\theta_1 = 86.722^\circ$, $\theta_2 = 0.86156^\circ$, and $\theta_3 = -0.25274^\circ$ in frame [A]. Of course, the relative angle rotations can also be measured by comparing the above three angles around different axes for each pose.

To study the performance of the proposed method, the measurement system is first calibrated by an autocollimator. The system is used to measure the relative rotations of the object around the x axis. Figure 5(a) shows the calibration result in a measurement range of 0–670 arc sec, which is mainly limited by the measurement limit of the autocollimator. The maximum deviation from the linearity of the measurement (or the calibration error) is about 1 arc sec. Since the object rotates only along one (x) axis, the fringe analysis method [10] can also be used for angle measurement in this case. The comparison (or calibration) of the angle measurement results between the proposed method and the fringe analysis method is shown in Fig. 5(b), which shows a much smaller calibration error of 0.005 arc sec and clearly shows the agreement between the proposed method and the fringe analysis method.

The measurement range around the z axis could be as large as 360°, which is not possible in any other currently available techniques. Figure 6 shows the calibration of the measured angle with the readout of a rotation stage within a 10° measurement range. Since the used rotation stage has only a limited resolution of 0.5°, the autocollimator is used for calibration as well by observing the object along the x axis in Fig. 1. Because of the range limit of the autocollimator, we only calibrate the system within a range of
tion error to the autocollimator is about axis. The up-left small figure shows that the calibration test of the proposed method within a measurement range of 10°; the inset shows a calibration error of ±0.005° by use of an autocollimator.

0.23° (or 828 arc sec), which is highlighted with a small circle in the figure. Note that the fringe analysis method cannot be used for measurement in this case since it is insensitive to rotations around the z axis. The up-left small figure shows that the calibration error to the autocollimator is about ±0.005°, which is not as good as that around the x (or y) axis in Fig. 5(a). This is mainly because of the small angle between the two facets of the object (α ~ 0.5°) in our current experiment prototype, which makes the system relatively sensitive to the phase map noise of the system. The phase noise will result in the deviation of the calculated propagation vectors from their true value; this will affect the accuracy of the measurement. The system performance could be improved by using a bigger α (α < 2αmax) between the two facets, reducing the phase noise, controlling vibration, improving the coherent length of the light source, or increasing the pixel number in the CCD camera. In practice, it is also possible to design a multiple-facet object so that more propagation vectors may be available, either to improve the measurement results by average and compensation or to guarantee a maximum measurement range around different axes by choosing different propagation vectors for analysis.

Although only mirrorlike objects are considered above, it is still possible to extend the proposed method for more complex and rough objects with speckle noise. Note that the FFT fringe analysis method does not work in this case since it requires a relatively clean phase map. However, the digital holography method can be used to reconstruct the object wave and extract the object spectrum at any specific plane. In this case the Fourier spectrum of the object will appear as an extended area with speckle noise, but not a well-defined point as in the case of a mirrorlike object. It is possible to get the propagation vector change information by directly evaluating the object spectrum in the frequency domain, but not the phase map. For example, the shift of the object spectrum can be measured by evaluating the cross covariance of the spectrum in the Fourier plane of the object. Although this method might be limited by the pixel resolution of the imaging sensor, a resolution of 0.3 mdeg (or 5 μrad) was reported in [11]. For a rough object, the light from the object will be scattered to different directions, so there are no absolute propagation directions of the object wave. If necessary, however, some featured points (such as the weight center or maximum cross-covariance point) of the object spectrum might still be used as the absolute propagation directions as in a mirrorlike object. Further work regarding this aspect is currently under way, and corresponding results will be reported in a forthcoming paper.

4. Conclusion

In conclusion, a new method for 3D angle measurement has been proposed based on the principle of propagation vector analysis. A multiple-facet object is used as an indicator of 3D rotations of a destination object, and the wavefields reflected from the multiple facets are recorded in a single hologram. Any small 3D rotation of the object will induce a change in the propagation vectors of the reflected waves in space, which can be extracted by analyzing the object spectrum of the recorded hologram and evaluating the phase maps of each wave component. Finally, 3D angle measurement is achieved by use of the proposed propagation vector analysis algorithm. Although the system used in our experiments is not fully optimized, the experiments presented above have clearly demonstrated the effectiveness of the method. The method may also be extended for complex and rough objects, but is not limited to mirrorlike objects.

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