PRACTICE SET FOR FINAL

Problem 1

Study completely the following functions (Find the domain, intercepts with the coordinate axes, symmetries, asymptotes, intervals of monotonicity and concavity, local max/min, inflection points and finally sketch the graph.)

$$f(x) = \frac{x+1}{x^2} \tag{1}$$

$$f(x) = \ln(x^2 - 4) \tag{2}$$

Problem 2

Compute the following limits

$$\lim_{x\to+\infty} \frac{\sqrt{4+x^2}}{9x} \tag{3}$$

$$\lim_{x\to 3} \frac{x^3 - 3x^2 - 2x + 6}{x^2 - 3x} \tag{4}$$

$$\lim_{x\to+\infty} \frac{\cos(x^3)}{x^2} \tag{5}$$

$$\lim_{x\to+\infty} \ln(x) - x \tag{6}$$

$$\lim_{x\to 0} \frac{\cos(3x) - 1}{\sin(7x)} \tag{7}$$

$$\lim_{x\to+\infty} x \operatorname{tg}\left(\frac{1}{x}\right) \tag{8}$$

$$\lim_{x\to+\infty} \left(1 - \frac{2}{3x}\right)^x \tag{9}$$

$$\lim_{x\to 0^+} \left(\sqrt{x}\right)^{3x} \tag{10}$$

$$\lim_{x\to 0} \frac{\int_0^{3x} e^t \cos(2t)\,\mathrm{d}t}{\operatorname{tg}(6x)} \tag{11}$$

$$\lim_{x\to 0} \frac{\int_0^{x^4} e^{t^2}\,\mathrm{d}t}{\int_0^{x^2} e^t t\,\mathrm{d}t} \tag{12}$$

Problem 3

Do the following equations admit any real solutions? How many?

$$\sqrt{1-x^2} = x \tag{13}$$

$$x^5 - x = 2 \tag{14}$$

$$x^4 - x + 2 = 0 \tag{15}$$

Problem 4
Compute the derivatives of the following functions

$$(16) \qquad f(x) = \frac{\operatorname{arctg}(x^2)}{e^{-x}}$$

$$(17) \qquad f(x) = \sqrt[4]{\ln(x)}$$

$$(18) \qquad f(x) = \arcsin\left(\frac{3x}{x+1}\right)$$

$$(19) \qquad f(x) = \operatorname{tg}^5(3x^2)$$

$$(20) \qquad f(x) = (x-2)^{\sin(x)}$$

$$(21) \qquad f(x) = \int_1^x |\sin(t)|\,\mathrm{d}t$$

$$(22) \qquad f(x) = \int_0^{x^2+x} \frac{\operatorname{arctg}(t)}{1+t^2}\,\mathrm{d}t$$

$$(23) \qquad f(x) = \int_{-x}^{x^2} e^{t^2} t^5\,\mathrm{d}t$$

Problem 5
A) Determine the equation of the tangent line to the graph of the function $y = \operatorname{arctg}(x)$ at $x = 1$.
Find all points where the tangent is horizontal.

B) Determine the equation of the tangent line to the curve $x^3 - \sin(y) + xy = 8$ at the point $(2, 0)$.
*)Are there any points where the tangent is horizontal?

C) Among all triangles inscribed in a semicircumference of radius 1 in such a way that one side coincides with the diameter find the one that has maximum area. Is there a triangle with minimum area?

D) We want to construct a box in the shape of a parallelepiped whose base length is 3 times the base width. The material used to build the top and bottom costs $10\$/ft^2$ and the material used to build the sides costs $6\$/ft^2$. If the box must have a volume of $50ft^3$ determine the dimensions that will minimize the cost to build the box.

E) Find the points on the ellipse $x^2 + \frac{y^2}{4} = 1$ that are furthest away from $(1, 0)$.

F) Air is being pumped into a spherical balloon at a rate of $1cm^3/s$. How fast is the radius increasing when the volume is $5cm^3$?

G) The top of a ladder slides down a vertical wall at a rate of 1 cm/s. At the moment when the bottom of the ladder is $10cm$ away from the wall, it slides away from the wall at a rate of $2cm/s$. How long is the ladder?

H) A ball is thrown in the air from a height of $1ft$ with initial velocity $\frac{1}{2}ft/s$. (We ignore air resistance). What is the maximum height it will reach? What would the inizial velocity have to be in order for the ball to reach the top of a building $50ft$ tall?

Problem 6

Compute the following integrals

$$\int_{-1}^{1} e^{-3x} - \frac{3}{1+x^2}\,\mathrm{d}x \tag{24}$$

$$\int_{0}^{1} \frac{2}{\sqrt{10-2x^2}} \tag{25}$$

$$\int x^2\cos(2x)\,\mathrm{d}x \tag{26}$$

$$\int x\cos(x^2)\,\mathrm{d}x \tag{27}$$

$$\int \frac{1}{x\sqrt{\ln(x)}}\,\mathrm{d}x \tag{28}$$

$$\int x\,\mathrm{arctg}(2x)\,\mathrm{d}x \tag{29}$$

$$\int e^{\sin(x)}\cos(x)\,\mathrm{d}x \tag{30}$$

$$\int \sin^5(x)\,\mathrm{d}x \tag{31}$$

$$\int \sec^2(x)\,\mathrm{tg}^4(x)\,\mathrm{d}x \tag{32}$$

$$\int \frac{6x+3}{x^2+x-1}\,\mathrm{d}x \tag{33}$$

$$\int \frac{\arcsin(x)}{\sqrt{1-x^2}} \tag{34}$$

$$\int xe^{-2x}\,\mathrm{d}x \tag{35}$$

$$\int \frac{x^3}{1+x^8}\,\mathrm{d}x \tag{36}$$

$$\int \frac{x^7}{1+x^8}\,\mathrm{d}x \tag{37}$$

$$\int \frac{1}{\sqrt{x}(1+\sqrt{x})}\,\mathrm{d}x \tag{38}$$

$$\int \frac{1}{1+\sqrt{x}}\,\mathrm{d}x \tag{39}$$