PRACTICE SET

Problem 1
Solve the following integrals

(1) $$\int (x-1)\sin x \,\mathrm{d}x$$

(2) $$\int \frac{\sin^3 x}{1+\cos^2 x}\,\mathrm{d}x$$

(3) $$\int \frac{\sec(\ln x)\tan(\ln x)}{x}\,\mathrm{d}x$$

(4) $$\int e^{\sqrt{x}}\,\mathrm{d}x$$

(5) $$\int \frac{5-2x}{x^3-4x^2+4x}\,\mathrm{d}x$$

(6) $$\int \frac{x^2+2x}{x^3-1}\,\mathrm{d}x$$

(7) $$\int \frac{\sqrt[3]{x^2}-1}{x}\,\mathrm{d}x$$

(8) $$\int (\tan^5 x+1)\sec^4 x\,\mathrm{d}x$$

(9) $$\int \arcsin(2x)\,\mathrm{d}x$$

(10) $$\int \sin^2 x\cos^2 x\,\mathrm{d}x$$

(11) $$\int \frac{e^x}{\sqrt{1-e^{2x}}}\,\mathrm{d}x$$

(12) $$\int \frac{\ln(\arctan x)}{1+x^2}\,\mathrm{d}x$$

(13) $$\int e^{\frac{x}{2}}\sin x\,\mathrm{d}x$$

(14) $$\int \sqrt{1-4x^2}\,\mathrm{d}x$$

(15) $$\int x^2 e^{-3x^3}\,\mathrm{d}x$$

(16) $$\int (x^3-1)\ln x\,\mathrm{d}x$$

(17) $$\int x\arctan(1+x)\,\mathrm{d}x$$

(18) $$\int_1^{\infty} \frac{1}{2x^2+x-1}\,\mathrm{d}x$$

(19) $$\int_0^{\infty} \frac{1}{5+x^2}\,\mathrm{d}x$$

(20) $$\int_0^{\infty} e^{-x}\sqrt{e^{-x}+3}\,\mathrm{d}x$$

(21) $$\int_0^{\frac{1}{2}} \frac{1}{x\ln^2 x}\,\mathrm{d}x$$

Problem 2
1) Find the area between the curves $y = x - x^2$ and $2y + 1 = x$.
1) Find the area between the curves $y = |x| - 1$ and $2y = x$.
1) Find the length of the curve given by the graph of the function $y = \frac{x^4}{8} + \frac{1}{4x^2}$, $1 \leq x \leq 2$.
2)Find the volume of the solid obtained by rotating the region between the curves $y = x^3$, $y = \sqrt{x}$ around the line $x = 1$. Same question but rotating around $y = 1$
3)Find the volume of the solid obtained by rotating the region between the curves $y = e^{-x^2}$, $y = 0$, $x = 0$ and $x = 1$ around the y axis.

Problem 3
1)A tank has the shape of the solid obtained by rotating the region between the curves $y = e^x - 1$, $x = 0$ and $y = 1$ around the y axis. The tank contains a liquid with density ρ kg/m^3. Find the work required to empty the tank by pumping all the liquid to the top.
2) Determine the force due to hydrostatic pressure on the flat vertical side of a tank which has the shape of the region enclosed by the curves $y = 4$ and $y = x^2$ if the liquid contained has density ρ kg/m^3. (The acceleration of gravity is g m/s^2, you can leave the constants without substituting their numerical value).

Problem 4
1) Determine the coordinates of a point R in the x axis in such a way that the triangle PQR is isosceles, where $P = (1, 2)$, $Q = (-3, 1)$. Compute its area and the amplitude of its internal angles. How many points satisfy that condition?
2) Determine the coordinates of a point R in the x axis in such a way that the triangle PQR is a right triangle, where $P = (1, 2)$, $Q = (-3, 1)$. Compute its area and the amplitude of its internal angles. How many points satisfy that condition?
3) Determine the coordinates of a point R in such a way that the triangle PQR is equilateral, where $P = (0, 1, 2)$, $Q = (-3, 0, 1)$. Compute its area. Find the equation of the plane containing it. How many points satisfy that condition?
4)Find a unit vector $\vec{v}$ perpendicular to the plane passing through $A = (1, 0, 0)$, $B = (-1, 1, 2)$ and the origin. Find the equation of the line parallel to $\vec{v}$ and passing through the point $C = (2, 1, 1)$.
5) Are the planes $2x - 2y = z$ and $x - y + z = 2$ parallel?If not, find the angle between them.
6) Find the equation of the line passing through the origin and perpendicular to the plane $x + 2y = z$.
7)Find the equation of the plane passing through the center of the sphere $x^2 + y^2 + z^2 - x + 2y - 8z = 0$ and containing the line $-x + 3 = 2 + y = 3z$.
8)Find the equation of the sphere passing through the origin and with center $C = (1, -1, \sqrt{2})$. Find the equation of the plane tangent to the sphere at the origin.