Functions: Introduction and Examples

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Calculus 1

Learning Objectives

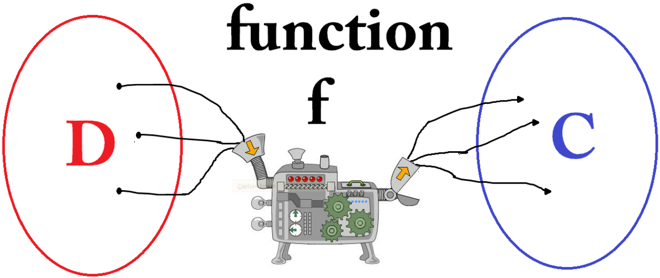
By the end of this lesson you will be able to...

- Define functions and the associated concepts of Domain and Codomain
- Classify different types of elementary functions
- Compute the domain of such functions

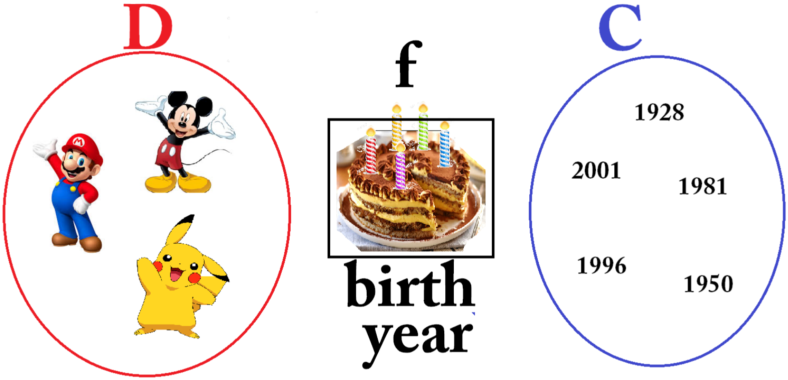
Table of contents

What is a function?

- A RULE that associates to each Input only 1 Output
- Input = Domain (D)
- Output = Codomain (C)



Examples of functions



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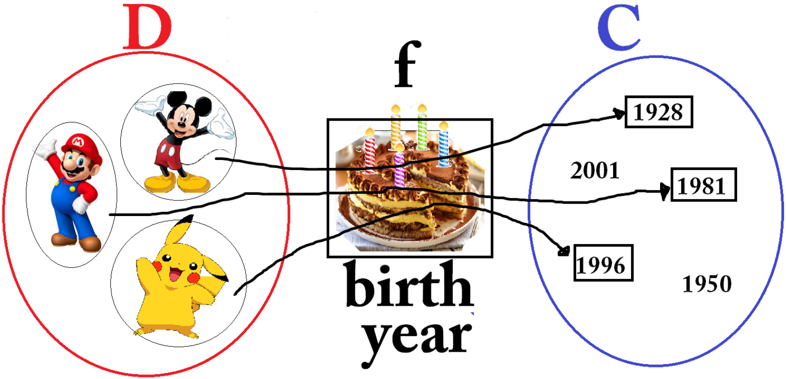

Functions of a Real variable

- Domain $\subset \mathbb{R}$
- Codomain $\subset \mathbb{R}$
- GRAPH representation in the plane

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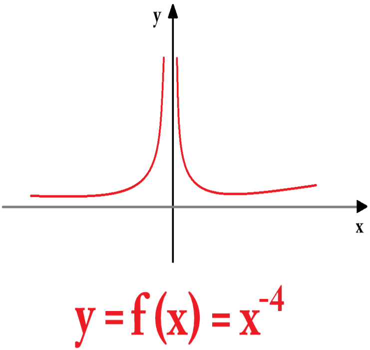

Polynomial Functions

Definition

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$$f(x) = 2x - \frac{5}{3} \quad \text{degree 1}$$

$$f(x) = x + \sqrt{2}x^3 \quad \text{degree 3, not ordered}$$

$$f(x) = 3x^{-2} + 4x^5 \quad \text{NOT a polynomial! WHY?}$$

Special Polynomials: Linear

LINEAR POLYNOMIAL=DEGREE 1

Special notation: $\mathbf{y} = \mathbf{mx} + \mathbf{q}$

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- m is the SLOPE
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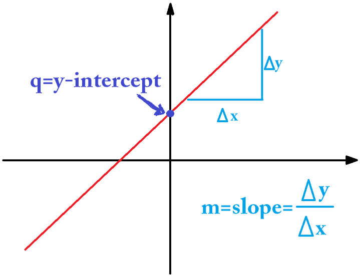

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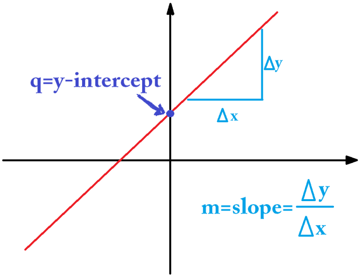

Discussion Question: Can we represent all lines through polynomial functions?

Special Polynomials: Quadratic

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Special notation: $y = ax^2 + bx + c$

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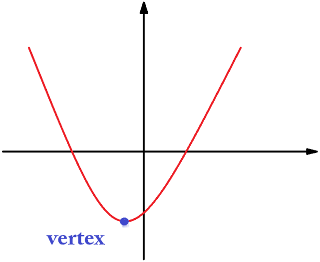

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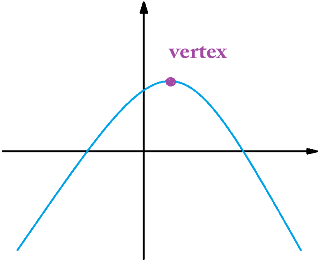

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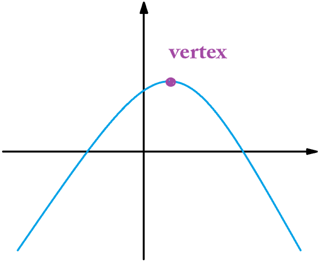

Quadratic Formula

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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What if we combine RATIONAL and IRRATIONAL Functions?

$$f(x) = \sqrt{\frac{2x^3 - 5x^5}{4x^2 - 3x + 1}}$$

Q: Can you find the DOMAIN of this function?