Intro to real business cycles:

Approach 2 ways: historical vs a-historical

Historical: failings with the other ways of explaining business cycles. Unpalatable assumptions, not great forecasting ability.

This approach would be better motivated if we had already studied business cycle theory.

A-historical: we have a model for the aggregate economy (like growth models), and some assumptions about optimizing behavior, clearing markets, etc. that we like. Now let’s see if we can use these tools to explain these fluctuations that we observe in the economy.

Elements of RBC models:

1. Clearing markets. There is never any disequilibrium in the markets, of the type discussed in Barro and Grossman (an approach that you won’t see taught this year, but might see referred to at some time). Thus RBC models are sometimes called “equilibrium” business cycle models. This means that no agent is ever constrained from taking the market price. In particular, it means that there is no such thing as “involuntary” unemployment.

2. Optimizing behavior on the part of all agents. (of course)

3. No nominal rigidities -- money will not matter for output (that's why they are called real business cycle models). This assumption is also called “flexible prices.”

4. Appeal to technology shocks. Since there has to be some stuff that moves the economy around, RBC models tend to look toward shocks to the production technology (or sometimes to tastes) to produce cycles.

Business Cycle facts to be explained:
See handout from the King and Rebelo ('resuscitating') handbook [first look at pictures, then at tables.]

Things to note:

1) Volatility

Consumption of non-durables is less volatile than output
Consumption of durables is more volatile than output
Investment is much more volatile than output (three times!)
Government expenditures are less volatile than output
Hours worked has same volatility as output

Employment is as volatile as output, while hours worked per employed person is less; thus most variation in total hours stems from changes in employment, not hours per worker.

Labor productivity (output per person-hour is less volatile than output)
The real wage is much less volatile than output.

2) Co-movement

Most series are pro-cyclical. Hours worked (N in their table) moves particularly closely with output. Wages, government expenditure, and capital stock have almost zero contemporaneous correlation with output.

Things to note:

1) Investment is more variable than other parts of GNP

2) Everything varies together: consumption, GNP, investment

3) Hours are very pro-cyclical. productivity is less variable and less correlated with output, but it is still pro-cyclical.

Approach: start with a growth economy that allows for shocks to technology but has inelastic labor supply. See what behavior you can get. Then allow for inter-temporal substitution of labor. This will get big movements in hours.

Ramsey model with shocks to productivity
We begin by exploring the effects of productivity shocks in models that we know and love. This will give us a baseline against which to consider the effects of adding various twists to the models.

We do the model in discrete time rather than continuous time, as was done before. This makes it much easier to specify shocks.

We also just do the social planner's problem.

Population constant at one, everything is per capita

Production function: $y_t = v_t k_t^\alpha$

$v_t$ is the productivity shock.

We assume that capital fully depreciates at the end of every period, so the difference equation for the capital stock is just

$$k_{t+1} = v_t k_t^\alpha - c_t$$

The first order condition relating consumption in periods $t$ and $t+1$ is the usual sort:

$$u'(c_t) = \frac{1}{(1+\theta)} E_t \left[ \alpha v_{t+1} k_{t+1}^\alpha \times u'(c_{t+1}) \right]$$

This has the usual logic: the left hand side is the cost of giving up one unit of consumption in the current period. The right hand side is the expected benefit (discounted).

Note that the marginal product of capital that is relevant to deciding between consumption in periods $t$ and $t+1$ is the marginal product in period $t+1$. That is, after production has taken place in period $t$, all of the capital goes away. Now you have your output that you can consume or invest. If you invest a marginal unit, you will get extra output next time equal to the marginal product of capital next period (which is not known now).

To make things tractable, we will assume log utility (so that $u'(c) = 1/c$). Note also that $\alpha$ and $k_{t+1}$ can be passed through the expectation sign:

$$\frac{1}{c_t} = \frac{\alpha k_{t+1}^{\alpha-1}}{(1+\theta)} E_t \left[ \frac{v_{t+1}}{c_{t+1}} \right]$$
So this is a pretty nasty beast, and usually we don't try to solve these things explicitly for consumption, but rather just describe the dynamics of the system. In this case, however, we can get an exact solution for $c_t$ as a function of $k_t$ and $v_t$.

We will state the solution, then show that it satisfies the first order condition: solution is to save a constant fraction $(\alpha/(1+\Theta))$ of output, and to consume the rest. So:

$$c_t = \left(1 - \frac{\alpha}{1+\theta}\right) v_t k_t^\alpha$$

Alternatively

$$\frac{v_t}{c_t} = \frac{1}{\left(1 - \frac{\alpha}{1+\theta}\right) k_t^\alpha}$$

Note that the rule says that consumption does not depend on the expected interest rate. This is the kind of result that we always get with log utility.

To show that this rule is indeed optimal, we will show that it satisfies the FOC. Substitute in for $c_t$ and for $c_{t+1}$:

$$\frac{1}{\left(1 - \frac{\alpha}{(1+\Theta)}\right) v_t k_t^\alpha} = \frac{\alpha k_{t+1}^\alpha}{\left(1 - \frac{\alpha}{1+\Theta}\right) k_{t+1}^\alpha}$$

since fraction $\alpha/(1+\Theta)$ of output is saved, the difference equation for capital stock is

$$k_{t+1} = \frac{\alpha}{1+\theta} v_t k_t^\alpha$$
substituting this into the FOC:

\[
\frac{1}{(1+\theta)}v_t k_t^\alpha v_t k_t^\alpha = \frac{\alpha}{(1+\theta)}v_t k_t^\alpha v_t k_t^\alpha = \frac{1}{(1+\theta)}v_t k_t^\alpha
\]

so indeed, the rule for consumption satisfies the FOC.

So given this, we know the stochastic process for capital and output.

\[
k_{i+1} = \left(\frac{\alpha}{1+\theta}\right) v_t k_t^\alpha = k_{i+1}^\alpha v_t^\alpha y_i
\]

raise both sides to the $\alpha$, and then multiply by $v_{t+1}$

\[
y_{i+1} = v_{t+1} k_{i+1}^\alpha = v_{t+1} \left(\frac{\alpha}{1+\theta}\right)^\alpha y_{i}^\alpha
\]

taking logs:

\[
\ln(y_{i+1}) = \alpha \ln\left(\frac{\alpha}{1+\theta}\right) + \alpha \ln(y_i) + \ln(v_{i+1})
\]

Which says that output will follow a mean-reverting AR(1) process. What is interesting about this? The point is that white noise shocks are propagated in this model. That is, a positive shock in one year will lead to effects on output over the course of several period.

Now note that the propagation mechanism in this model is through the capital stock: when there is a good shock, extra capital gets built, and it stays around for a while. More concretely: when extra capital it built, it raises the wage in the next period, and so leads to more capital being built in that period, etc.

This is only one sort of propagation mechanism -- and probably not one that anyone takes too seriously. The point is to demonstrate the idea that the economy is bombarded with exogenous
shocks, and that the structure of the economy determines some propagation mechanism.

**Labor supply as a way to beef up the response to technology shocks**

Looking back at the facts, we see that there are big co-movements in GNP and total hours (or average weekly hours). Prescott reports the correlation of deviations in trend for the two of these as .85.

So we are going to want to build this movement of hours into the RBC model.

So why should hours vary over the business cycle? To see why, let's look at our basic intertemporal optimization problem, this time allowing for labor supply to be a choice variable.

Let's solve the individual's problem, taking interest rates and wages as exogenous.

define $N$ as the amount of leisure the a person consumes per period. We get utility from consumption and leisure, so the instantaneous utility function is $U_t = U(C_t, N_t)$.

Each period, the person has an endowment of time (say the total number of hours they are awake), which she can divide between leisure and work. Call the total endowment 1, and so time spent working is $(1-N_t)$. There is some exogenous wage, $w$, so total labor income in period $t$ is just $w_t(1-N_t)$.

There is some interest rate, $r$, which we will just hold as constant for now. There is also some discount rate, $\theta$.

So the individual's problem is

$$\max \sum_{t=0}^{T-1} U(C_t, N_t)(1+\theta)^t$$

s.t.

$$\sum_{t=0}^{T-1} C_t(1+r)^t = \sum_{t=0}^{T-1} (1-N_t)w_t(1+r)^t$$
so we set up the old Lagrangian:

\[ L = \sum_{t=0}^{T-1} U(C_t, N_t)(1 + \theta)^{t'} + \lambda \left( \sum_{t=0}^{T-1} (1 - N_t)W_t(1 + r)^{t'} - (1 + r)^t \right) \]

to solve, we would just differentiate this with respect to the \( T \) different values of \( C \), the \( T \) different values of \( N \), and \( \lambda \). This would give us as many FOC's as unknowns, so we could solve. We will just look at the FOC's for \( C \) and \( N \) in periods 0 and 1 (obviously it could be any two periods).
combining (1) and (2) gives us the usual FOC for consumption:

\[
\frac{dL}{dC_0} = U_c(C_0, N_0) - \lambda = 0 \implies U_c(C_0, N_0) = \lambda
\]

2) \[
\frac{dL}{dC_1} = U_c(C_1, N_1)(1+\theta)^1 - \lambda(1+r)^1 = 0 \implies U_c(C_1, N_1) = \frac{\lambda(1+\theta)}{1+r}
\]

3) \[
\frac{dL}{dN_0} = U_n(C_0, N_0) - \lambda w_0 = 0 \implies U_n(C_0, N_0) = \lambda w_0
\]

4) \[
\frac{dL}{dN_1} = U_n(C_1, N_1)(1+\theta)^1 - \lambda w_1(1+r)^1 = 0 \implies U_n(C_1, N_1) = \frac{\lambda w_1(1+\theta)}{1+r}
\]

This says that if \(w_1\) is higher than \(w_0\), then you want higher marginal utility of leisure in period 1 -- and the way you get higher marginal utility of leisure is to have less of it. So you work more in response to higher wages.

We can apply the same intuition that we developed about intertemporal consumption smoothing to the question of intertemporal leisure smoothing. That is, the key question is the intertemporal elasticity of substitution -- the willingness of people to move consumption or leisure from one
period to another.

As with consumption, the key is the curvature of the utility function -- if there is rapidly declining marginal utility of leisure, then people will not readily substitute leisure from one period to another in response to higher wages. On the other hand, if there is fairly constant marginal utility of leisure, then in response to wage changes, people will be willing to shift leisure around -- and then you will get a big response of labor input to changes in the real wage.

We can also consider the effects of changing the interest rate on labor supply: raising the interest rate that holds between period 0 and 1 makes us want to work more in period 0 relative to period 1.

Point of all of this -- when you have a productivity shock, the wage goes up. This will cause people to intertemporally substitute.

Lucas example: how much extra do you have to pay people to move their vacations?

This first order condition is clearly the key to thinking about how labor input should vary over the business cycle: when there is a good shock, output goes up because people work more, and because people are more productive.

[aside here or elsewhere: we can get an even bigger effect if we allow for time-non-separable utility from leisure: say that the utility of leisure today depends on how much leisure you consumed yesterday (or last quarter, or last year, or whatever).

An example of such a utility function is the one used in Kydland and Prescott's model:

\[
\begin{align*}
    u(c,l) &= \ln(c_i) + \ln \left( \frac{1}{\eta} \sum_{t=0}^{\infty} l_{t+1} \left( 1 - \eta \right)^t \right) \\
\end{align*}
\]

current effective leisure is a weighted average of current and past leisure. With such a specification of utility of leisure, if there is extra-high productivity today, and regular productivity tomorrow, you may work more than usual today, but then less than usual tomorrow. So this will exacerbate the fluctuation-causing power of productivity shocks.]

Finally, we can also look at the “static first order condition,” derived from looking at FOCs (1) and (3):

\[
\begin{align*}
    U_d(C_0, N_0) &= w_0 U_c(C_0, N_0) \\
\end{align*}
\]

This just says that the marginal utility of consumption in a period has to be equal to the marginal utility of leisure in that period times the wage -- or else people would just work more or less.
We can think of all of these relations -- between consumption in adjacent periods, between leisure in adjacent periods, and between consumption and leisure in the same period -- arranged as follows:

\[
\begin{align*}
&c_0 \quad c_1 \quad c_2 \quad \text{etc.} \\
&| \quad | \quad | \\
&n_0 \quad n_1 \quad n_2 \quad \text{etc.}
\end{align*}
\]

It is clear that some of the restrictions are redundant -- that is, you do not need to know them to pin down all of the values of c and n. For example, if you satisfy all of the dynamic c and n FOCs, and just one of the static FOCs, then you can be sure that you are satisfying all of the other static FOC's. Or if you are satisfying the dynamic c FOCs, and all of the static FOCs, then you don't need to check the dynamic FOCs for n.

Let's look at the effect of intertemporal substitution on fluctuations in a series of models. We will also look at the effects of government spending in these models, for reasons that will become clear.

[General note: RBC models are generally so complicated that they cannot be solved analytically, but have to be simulated instead. Our approach to describing them will be to look at a series of simplified special cases, where the model can be solved analytically. This will aid us in getting an understanding about what happens in the full model].

Consider an economy where output is produced by labor input alone. There is no capital, and thus no means to transfer output from one period to another.

There is a production function that takes labor as the only argument:

\[
y = f(l) \quad f' > 0
\]

We begin by considering the case where labor is supplied inelastically: \( l = \bar{l} \). People get utility only from consumption: \( u = u(c) \).

Even though there is no capital, there can still be assets -- that is, promises to pay someone in period \( t+1 \) in return for a payment in period \( t \). Everyone in the economy is identical, so they will not trade any of the assets -- but we can still price the assets (that is, we can find the price, or interest rate, for which the asset will have zero excess demand).

Call \( r_{t+1} \) the interest rate on assets held between periods \( t \) and \( t+1 \).
Consider the case where there are changes in the amount of output that is consumed by
government expenditures (financed by lump sum taxes). People get no utility from government
expenditures. Consumption is
\[ c_t = f(l) - g_t \]

By writing down the first order conditions for consumption, we can find the interest rate in every
period.
\[ \frac{u'(c_{t+1})}{u'(c_t)} = \frac{1 + \theta}{1 + r_{t+1}} \quad \text{so...} \quad (1 + \theta) \left( 1 + \frac{u'(f(l))}{u'(f(l))} \right) \]

Now consider the effect of two different patterns of government spending: one in which the
government spends a lot in one period, the other in which the government has higher spending in
each period. What effects will these two policies have on interest rates?

Answer: the temporary increase in period \( t \) will raise the interest rate that holds between periods \( t \)
and \( t+1 \). (similarly, if it is expected, it will lower the interest rates between periods \( t-1 \) and \( t \)). The
idea is that the interest rate has to rise to make people want to consume less in period \( t \) relative to
other periods. In the case of a permanent increase, there is no effect on the interest rate.

[You might ask when the relevant taxes are being collected -- but of course, under Ricardian
Equivalence it does not matter. Suppose that the government decided to use debt to finance it's
temporarily high expenditure in period \( t \). Then it would have to issue IOU's in the amount \( g_t \), and
collect taxes \( g_t(1+r_{t+1}) \) in period \( t+1 \). The interest rate necessary to get people to want to hold \( g_t \) in
IOU's will be exactly the interest rate that would have prevailed if the spending had been financed
by contemporaneous taxes.]

Now forget about the government and consider the effect of productivity shocks. Let \( v_t \) be the
multiplicative productivity shock in period \( t \).
\[ c_t = v_t f(l) \]

What is the relationship between interest rates and productivity shocks? The first order condition
is just
\[ \frac{u'(c_{t+1})}{u'(c_t)} = \frac{1 + \theta}{1 + r_{t+1}} \quad \text{so...} \quad (1 + \theta) \left( 1 + \frac{u'(f(l))}{u'(f(l))} \right) \]

Higher productivity in period \( t \) raises consumption and lowers the marginal utility of consumption
-- thus lowering the interest rate that has to hold between periods \( t \) and \( t+1 \).
As an exercise: figure out the effect of changing the coefficient of relative risk aversion on the effect of productivity shocks on the interest rate.

Now expand the above model to allow for individuals to make a labor/leisure choice in each period. (It is still the case that there is no capital). let \( n_t \) be the amount of leisure in period \( t \), and \((1-n_t)\) be labor input. Utility is defined over \( c \) and \( n \).

Again consider the effect of government expenditures. Consumption is

\[ c_t = f(1-n_t) - g_t \]

For simplicity, consider the case where output is linear in labor input: \( f(1-n_t)=w(1-n_t) \)

The static FOC is \( w U_c = U_n \). We can see that the effect of of increasing \( g \) is that consumption and leisure will both fall, and that output will rise. (how to see this: think about if consumption stayed constant. Then leisure would have to fall, and the right side of the FOC would be bigger than the left. What if leisure stayed constant? Then the left side of the FOC would be bigger than the right...)

What about the effects on the interest rate? Here it will depend on whether it is a permanent or temporary increase. As we showed above, for a permanent increase, there will be no interest rate effect. For a temporary increase, there will be a rise in interest rates.

[note the simple structure of the model: static FOC pins down consumption in every period, and consumption determines interest rates]

Thus a temporary increase in government spending has induced a rise in output and in interest rates. This is the "neoclassical" model of fiscal policy's effects on the economy. This is an alternative to the Keynesian view of fiscal policy's effects on output, which goes through the channel of price stickiness. [Again, the idea of RBC models is to produce a model that predicts things that we actually see in the real world, but not have it rely on unpalatable assumptions.]

What about the effects of productivity shock?

Holding labor input constant, this will raise the wage (that is, it will shift out the labor demand curve). What will it do to consumption and leisure? Given the static FOC, \( v_t w u_c = u_n \), there are two effects: holding \( n \) constant for a moment, \( w \) will rise, but \( c \) will also, so \( u_c \) will fall. So the effect is ambiguous. Notice (here we are deriving intuition, not being formal) that the reason that we have this offsetting effect is because the productivity shock raised consumption. This has to be the case in the model with no capital, because there is no way to transfer consumption forward in time. More generally, it will be the case for permanent productivity shocks in a model where consumption can be shifted from period to period (by changing the saving rate). But in a model where consumption can be shifted around, temporary shocks to productivity will have an effect on wages, but not on consumption, and so to satisfy the FOC, leisure will have to fall -- this is the big
labor supply effect needed in RBC models.

Now note that we have not included capital in this model. Remember that the Ramsey model that we did had capital and optimal consumption (although there we had to do log utility or it would have been too hairy), but not labor supply. We can try to intuit what would happen if we had capital in this model. In such a case, a good productivity shock leads to more work, but instead of more consumption, we could have more investment. Then get the propagating effect from the capital stock.

To see this point, think about a model with an exogenous real interest rate. Think about a one-period shock that doesn't change the lifetime budget constraint, and so does not change consumption. Then \( w^*u_c = u_n \) holds, and \( n \) must fall when productivity rises. So output will go up both because of the productivity shock, and because of the labor supply response.

**Calibrating and testing RBC models**

The above exercises give some intuition about the different effects that are present in RBC models. But they have only looked at restricted cases. This is because, beyond these simple cases, RBC models are far too complicated to be examined analytically. The approach, rather, is to fully and as realistically as possible specify an "artificial economy," then simulate this economy and examine its properties.

[Note: RBC types tend to refer to their creations as "economies" rather than "models." (for example, Prescott labels a table "Cyclical Behavior of the Kydland-Prescott Economy.") ]

[See Prescott paper on the reading list for an example of this] The test to which RBC models are put is whether they can replicate various features of the actual data. Since there are so many parameters floating around the model, one might think it would be easy to fit a few aggregate numbers. To make it a fair test, then, the creators of RBC models try to take values for as many parameters as possible from other sources: generally studies of micro data, but also studies of non-business cycle frequency macro data.

For example, take the question of the exponent on capital in the production function. We could assume that the production function was cobbdouglas: \( Y = A K^{\alpha}L^{1-\alpha} \), and then we would use capital's share of aggregate income to give us a value for \( \alpha \).

Similarly, to calibrate the utility function from which we derive the labor/leisure choice, we need to know the fraction of time that people spend in each activity -- which we can get from micro surveys.

Given all of these exogenously determined parameters, one then puts together the optimal decision rules of economic agents. In the Prescott paper, the only state variables are the capital
stock and the technology shock, so all decisions are functions of these (note that is utility from leisure is not time separable, then past leisure will also enter these functions).

Technology is one of the state variables because it is mean reverting

\[ z_{t+1} = \rho z_t + \epsilon_{t+1} \]

capital evolves normally...

\[ k_{t+1} = (1-\delta)k_t + y(k_t,z_t) - c_t \]

For the decision variables, we solve for the optimum choices, where of course these choices incorporate rational expectations of future values of everything.

(1) \( c_t = c(w_t, r_t, k_t, E(...)) \)

(2) \( n_t = n(w_t, r_t, k_t, E(...)) \)

the wage and the interest rate are determined from the production fn:

(3) \( w_t = w(k_t, z_t, n_t) \)

(4) \( r_t = r(k_t, z_t, n_t) \)

and of course expectations have to be consistent with today's choice variables:

(5) \( E_t (\text{stuff}) = f(\text{stuff}_t) \)

equations (1) through (5) are all solved simultaneously, giving values of all of the choice variables.

Of course, we usually can't write down exact solutions for all of these things, but we can approximate them in various ways...

Note that the parameters that we took from micro data will be present in the functions (c(), n(), etc.) that we derive. Also present will be the "free parameters," about which we had no a priori information.

Then, when everything is specified, we put it in a computer and let it rip .. that is, we generate new values of \( \epsilon \) every period, and let the economy roll along. Then we go back and look at the data that has been generated, and see how it compares to what we see in the real world.

If there are free parameters in the model, we can now try to change some of them to improve the
fit of the model (say in some systematic fashion, as we would when "climbing a hill" in maximum likelihood estimation).

The results of this model fitting exercise are usually summarized in a side by side comparison:

[See table in the King and Rebelo handout]

The question of what sort of metric should be applied to this sort of comparison is still not settled. That is, we do not have a statistical theory for this sort of stuff the way that we do for formal econometrics. We thus have no way of judging what we should conclude about the veracity of the model from our ability to fit these moments.

In any case, there is one key place where this model falls down -- as do most other RBC models -- and this is on the correlation of output with productivity. It is much higher in the model than in reality. Recall this is because the key channel to higher output is productivity shock ==> wage rise ==> higher labor supply ==> higher output. If labor supply varies for non-productivity reasons (as Keynesians would argue that it does), then one would expect no correlation between output and hours on the one hand and productivity on the other.

On the subject of whether it is cheating to allow the free parameters to vary, Prescott admits that he would like better measures. The odd title of his article, “Theory Ahead of Business Cycle Measurement,” is a reference to a famous book review. The book being reviewed, Measuring Business Cycles (1947) by Arthur Burns and Wesley Mitchell, was a huge compendium of measures of the cyclical properties of hundreds of economic time series (showing, for example, that pig iron production tended to lead or lag behind aggregate output). The review, by Tjalling Koopmans, was entitled “Measurement Without Theory,” and argued that it was silly to gather so much data without a unifying framework in which to analyze it. Thus with his title Prescott seems to be claiming that the situation is now reversed.

Another problem with the fit of the model presented here is that if you looked at the hours more carefully, you would see that it does not match reality in the following way: in the model, variation in hours comes from everyone reducing labor input by a little (everyone has to do the same thing, since they are all representative Robinson Crusoes). In the real economy, by contrast, variation in total hours is mostly accomplished by some people moving between full time employment and unemployment, while employed people work a fairly constant number of hours. If \( H \) is total hours, \( h \) is hours per worker, and \( N \) is the number of workers, then it is true that:

\[
\text{Var}(\ln(H)) = \text{Var}(\ln(N)) + \text{Var}(\ln(h)) + 2 \text{Cov}(\ln(N), \ln(h))
\]

Hansen (1985) shows that 55% of the variance in \( \ln H \) is due to the variance of \( \ln N \), 20% is due to the variance of \( \ln h \), and the remaining 25% comes from the covariance term.

This problem is solved by Gary Hansen as follows....
Start with the production function mapping hours of work into "hours of labor services." We have been assuming implicitly that this was just linear -- a straight line from the origin. So we could just graph the marginal benefit of working as a horizontal straight line. This will just be the marginal utility of consumption times the wage. Note that for an individual considering how much to work on a given day (or month, or short period of time), the extra consumption due to working more will be spread over the entire lifetime -- and so the marginal utility of extra income will not decline with working more in a given day. Thus the marginal benefit of working will just be a horizontal line.

Against this we would put the marginal utility of leisure increasing with the number of hours worked, and find the number of hours the person would choose. We would then consider different wages (which would shift up the marginal benefit of working), and get the individual's labor supply curve -- which would be the same for the individual as for the economy in a RBC world.

But we could think that there was a different mapping of hours supplied to productive work: say there is some fixed, unproductive cost (commuting, dressing up, having coffee and gossiping). So the graph showing the marginal benefit of working is a step function, where the wage affects the height of the step part.

Now there are several possibilities: if the marginal utility of leisure is always higher than the marginal benefit of working, they you never work. If they cross, it would be natural to conclude that the person would want to work the number of hours given by the point where they cross -- but this could be wrong! Because in addition to checking interior maxima, we also have to check end-point values. In this case, we have to check whether the individual might not be better off working zero hours.

To see this, graph the contribution to total lifetime utility of a day's labor and leisure as a function of hours worked. (This is just the sum of the two pieces: utility from leisure and consumption). It slopes down initially, then up to a local max (at the crossing of the marginal benefit curves derived above), then down again. Changing the wage will not shift the left-most point on the curve, but it will shift the rest of the curve up and down. Depending on the wage, either zero labor supply or the interior local max will be chosen -- and for one level of the wage, individuals will be indifferent between the two. Call this wage $w^*$. 

Now draw the labor supply curve for the economy (made up of identical individuals). Labor supply is zero below $w^*$, then jumps up, after which it is upward-sloping.

Assume that labor demand is generated the usual way by firms hiring labor such that the marginal product is equal to the wage -- it slopes down. What do we make of a situation where demand crosses supply at $w^*$? At this wage, workers are indifferent between working and not working -- and so we can imagine a variety of devices that make sure that just the right fraction of them decide to work such that supply equals demand.
Now let labor demand shift around due to productivity shocks -- the wage will remain constant at \( w^* \), while the quantity of labor shifts around. Further, quantity shifts will occur due to movement of people in and out of working, rather than adjustments of hours! This is the basis of the Hansen model.

These fixed costs of going to work are an example of a non-convexity, in this case in production. We could also introduce non convexities in preferences: working an infinitesimal number of hours will give you much less utility than working zero hours. For example, if you work even a little, you can't go to Florida, have to wear a tie, etc. Non-convexities in preferences would produce the same results as the story presented here.

[Aside: Looking beyond business cycles, some sort of non-convexity may be the explanation for the way we do retirement: all at once. Most models (with declining productivity or increasing disutility of working), would say that you have a gradual retirement since you set the wage equal to the marginal utility of leisure. As you get older, wage falls gradually (or the marginal utility of leisure rises gradually), and so, without a non-convexity, we should observe people gradually increasing leisure.]

[behavioral economics aside]

Data from taxi driver trip sheets. Record all the trips taken, how long the cab driver worked, how much earned.

- Taxi drivers are a good subject for the study for two reasons:

1) they have control over their own hours – they can decide when to go home. (For regular workers, there are coordination issues).
2) their wage varies from day to day due to weather and other demand factors.

- Wage is calculated by dividing all fares collected in an hour by number of taxis working that hour. Two key findings:
  - hourly wage is variable across days. The average hourly wage ranged from $13 to $19.
  - wages positively correlated within days. If the hourly wage is higher early in the day, it is likely to be high later in the day as well. This is because demand for taxi rides is higher on rainy days, etc.

Key test is to see how hours vary with the wage.

Finding: drivers work less on high wage days!
If drivers worked a fixed number of hours per day (rather than working less when wage is high), holding average hours constant, they would raise their wage income by 5%.

If they worked more on high-wage days (with an elasticity of 1), again holding their total hours constant, then they would raise their average wage by 15.6%.

What should the elasticity be? We can figure this out by holding the marginal utility of income constant (since we are looking at a single day)

\[
\text{Max } U_y w (1-N) + N^{\lambda} / (1-\sigma)
\]

get \( \frac{d \ln(N)}{d \ln(w)} = -1/\sigma \)

But note that \(N\) was leisure – the elasticity of the wage will be different (but not much, if they are roughly equal)

So why do they have the wrong sign?

- authors suggest “daily income targeting” as a way of dealing with self control issues
- don't have to save money from good days to bad
- gives an easy rule for when to quit. Else would be temptation to say that any given day was a bad one. Similar to writing a set number pages per day!

[end of behavioral aside]

Hansen Model:

People are restricted to just work either full time or not at all -- that is, people just operate on the extensive margin.

Households make their labor supply decisions by choosing lotteries: in return for a fixed wage payment, the household participates in a lottery where it will work (full time) with probability \(\alpha\). (Adding lotteries can only make the individual better off, since the individual could always choose \(\alpha=1\) or \(\alpha=0\).) It turns out that the payment required to get a household to raise it's probability of working is constant: that is, to double \(\alpha\), you just have to pay twice as much. Thus, from the point of view of the economy as a whole, the labor supply curve is perfectly elastic.

Question: are the unemployed guys happy in the Hansen model? Answer: in the Hansen model they are happier than the working people, since they get the same wage and higher leisure. Clearly this doesn't match real life, but it's not supposed to -- rather, it is a convenient way of implementing the non-convexities story presented above.
Hansen takes this piece and combines it with a standard RBC model. He then does the same sort of simulation exercise that Prescott presents.

Hansen's results: his model produces more variation in bodies (and less in hours per person) than does a regular RBC model.

One problem with the Hansen story is that cyclical unemployment seems to be concentrated among a small group of people, not spread uniformly over the whole economy. [MRS give cites in conclusion].

Lucas and Rapping (JPE 77, Sept 1969)

This article is the start of the freshwater view of "unemployment." What does it mean to be unemployed?

-- definition of unemployed; how the numbers are collected.

-- observation that in the long run labor supply has not changed much in response to big changes in the real wage: this has been taken to indicate that the labor supply curve was vertical, and thus led to the conclusion that any temporary drops in employment were evidence that employment was "off the labor supply curve," that is, that people were involuntarily unemployed.

=> L&R pointed out that long run constancy of labor supply did not mean that there would be no short run substitution

L&R say measured "labor force" is the amount of labor which would be forthcoming at perceived "normal" real wages. In other words, people who are unemployed (beyond frictional unemployment) are people who view the wage at which they could currently work as low, and so sit out working (if their wage were permanently low, they would move occupations or locations or search for work). So unemployment is voluntary, but that does not mean that unemployed people are happy. But what they are unhappy about is not that they cannot work, but that the available wage is lower than they would like it to be.

Note that L&R are right, in the sense that even when unemployment is very high, there are usually some jobs (like delivering pizza) that people will not take.

But note that to apply the L&R story to RBC models, we have to go further, and say that people are willing to sit out of working when the wage falls just a bit. In RBC models, tech shocks are not sufficiently huge (std dev of real wages in these models is just 1 or two percent -- this is of different magnitude than a guy who gets laid off from a steel mill job paying $20/hr saying he is unemployed even though he could deliver pizza for $5/hr).

As a historical note, the L&R story, further developed by Lucas (understanding business cycles, 1977?) was not originally designed as part of a "real" model, but as part of an equilibrium story for a model in which nominal rigidities played a role. In his story (called the "Lucas misperceptions model"), which Peter may teach you, increases in the money supply make people think that there
real wage is higher, and so they work harder. The model is an equilibrium model (vs the standard Keynesian model) in that no one is constrained: during a recession, for example, people do not work because they think that the real wage is low, and so they do not want to work.

The part of the L&R story that focuses on people not immediately finding new jobs or moving in response to shocks in the available jobs is studied under the heading of "sectoral shifts." [which B&F discuss in ch 7].

For example, Lillien shows that there is a positive correlation between the aggregate unemployment rate and an index of sectoral variance: the std of employment growth across sectors. (see also Brainard and Cutler, QJE 1993)

Mankiw, Rotemberg, and Summers try estimating a utility function with intertemporal substitution of leisure and consumption. Get pretty wacky results. Big problem is that, while consumption and leisure move in opposite directions over the course of the business cycle, there is not much movement of the real wage.

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What about the cyclicality real wage? This is an old and difficult measurement problem.

-- How Keynes saw it: sticky nominal wage leads to counter cyclical real wage.

-- measured average wage is not very cyclic.

-- quality problem: composition effect. What we really care about is the wage that an individual faces. Over the course of the business cycle, there may be changes in the quality of the average worker (and we only observe wages for people who are working). For example, if low wage workers are the ones laid off during recessions, then even if individual wages were not cyclical, the average wage would be counter cyclical.

Stuff on cyclic of real wage. Keane, Moffitt, and Runkle (JPE 96:6, 1988) look carefully at the question of the cyclicality of the real wage. Find that is pro-cyclical, but not very largely so.

**Evaluation of RBC Models**

First problem with the RBC paradigm is the story about labor markets. The RBC story is that, first, there are variations in the real wage (and interest rates) over the business cycle and that, second, these variations cause intertemporal substitution in labor input, producing variations in employment.

The evidence against this labor market story is first that the real wage is not all that cyclical, but primarily that what happens in the labor market over the business cycle doesn't look like and equilibrium outcome. That is, people getting their houses re-possessed because they are out of work and can't make mortgage payments do not really seem to be taking leisure.
So let's forget about the labor/leisure choice as a way to understand unemployment, and see what if there is other useful stuff left in the RBC framework.

The other big part of the RBC model is the technology shocks. Take a step back and ask, why do we need "shocks" in a model.

Idea is that model is a big set of difference equations (many of which incorporate expectations, of course). If there are no shocks, can such a system continually produce cycles? The answer is yes...

Aside one dynamical systems: The old view about shocks was that you needed some kind of external shock to impinge on the dynamical system in order to stop it from moving to some steady state (or growth path). Without shocks of some kind, it was thought that there could be no behavior that looked like fluctuations. Recently, it has been pointed out that even deterministic (that is, not subject to outside shocks) systems of difference or differential equations can display all sorts of erratic behavior -- for example moving smoothly along for a while, then jumping to some new state in an apparently random manner. This work has gone under the rubric "chaos" theory -- the dynamics are described as chaotic because they look like there is no order... Standard examples: the weather, etc. The department's local chaos connoisseur is Oded. [You can also look at Gleick's book for a low tech introduction]

Should we take chaotic dynamics seriously as far as business cycles go? I don't think so, but I'm not sure why.

So let's say that we do need shocks to get fluctuations. Shall we take the RBC story of technology shocks seriously, and if not, where will the shocks come from?

Back in the section on growth theory we showed that if you start with a general production function, \( Y = A \cdot F(K, L) \), where \( F() \) is CRS, then one can derive the "growth accounting" equation.

\[
\frac{dY}{Y} = \frac{dA}{A} + (1 - \alpha) \frac{dL}{L} + \alpha \frac{dK}{K}
\]

where \( \alpha \) is just capital's share of output.

Since \( Y, K, \) and \( L \) are known each year, this equation can be used to back out the Solow residual, \( Da/A \), for every year.

When we first looked at Solow residuals, we focussed on long run averages in order to back out the long run growth rate of technology. But one can also look at Solow residuals on a year to year basis. This is done in figure 1 in the Plosser article. The solow residual averages .8%, with a standard deviation of 1.9% - the range is from 4% to -3.5%. In Plosser's plot, the two biggest negative solow residuals come in 1974 and 1982 -- which of course correspond to the two biggest
postwar recessions.

What are these technology shocks? Do they mean that we really forget how to make stuff? RBC answer is that they can be things like oil prices in 1974 (RBC model is for a closed econ, so change in price of an imported input cannot be explicitly modelled. But the idea is that trading can be viewed as part of the production process, and so our technology did get worse). 1982 is harder to explain. (But technology shocks can be broadly defined to include laws, etc.).

The other thing Plosser does is take these actual productivity shocks and feed them through a RBC model, to see what patterns of output, hours, etc. they produce. He finds that the derived patterns of these variables correspond pretty well to what we actually see in the economy. [Note that this differs from the usual RBC "moment matching" exercise, in which the moments, but not the actual generated data, from a RBC model are compared to those from the actual economy.]

Does this story about shocks make sense? Reasons to think not:

1. Except for oil in 1974 and 1979, it is hard to put labels on the episodes of negative (or highly positive) Solow residuals. One would think that if these things are so important, we should have names for more of them. We would expect to see them discussed on the news, etc. Further, since the model assumes that people in the economy know about the stochastic process for these things, we would hope that economists would know about them too. [RBC types will sometimes talk about weather -- and there was even some work looking at the relation between measured severity of winter and business cycles. But it hasn't really caught on.]

2. It's hard to think of what process would be generating these tech shocks. If we think of aggregate tech growth as being the sum of lots (say thousands) of independent attempts to improve technology, each of which might or might not succeed, then we would expect the law of large numbers to guarantee that about the same number would succeed each year. Also, no story about inventions will give the negative solow resid. [on the other hand, defining technology more broadly -- for example changes in government regulations (like the passage of the Clean Air Act) -- there could be just a few shocks per year.

3. They explain too much!!! Mankiw presents a version of the picture that Plosser presents, but he adds to it a line showing output growth. Shows that not only is there a correlation between output growth and the measured Solow residual, but that the two fit incredibly closely. This raises the stakes in the sense that, either we argue that there are no other explanations for fluctuations except tech shocks (which Prescott might believe, but other RBC types might disagree), or we accept that other things that cause output movements will also cause the measured Solow residual to move.

Mankiw gives the example of world War II: from 1939 to 1944, measured Total Factor Productivity (that is A), rose 7.6% per year. We are pretty sure that this was not due to some exogenous technological shock.
But if all of these objections are right, there is still the question of why the solow residual is so correlated with output.

The leading explanation for this result is "labor hoarding." The labor hoarding story is pretty simple: firms face costs of hiring and firing workers. So when the amount that the firm wants to produce changes, the firm may, rather than fire workers, simply work them less hard. The result is that, if you could measure effort, it would be higher during a boom than during a recession. This in turn means that total labor input -- hours times effort -- is understated in a boom and over stated in a recession. Assuming the production function is $Y=A*F(K,E*L)$, the right growth accounting equation is

$$\frac{dY}{Y} = \frac{dA}{A} + (1-\alpha)\frac{dL}{L} + (1-\alpha)\frac{dE}{E} + \alpha\frac{dK}{K}$$

If $E$ is procyclical but not measured, the solow residual will appear to be procyclical even if it is not. [This may be part of the explanation for high productivity growth in the late 1990s -- there was labor dis-hoarding, ie everyone was working (temporarily) very hard.]

Note that the labor hoarding story says that the pro-cyclical productivity which we saw when we began looking at business cycles is just a measurement problem.

McCallum, p31 -- timing of corrs as evidence of labor hoarding?

So this is the non-RBC story for why the measured Solow residual is so procyclical. The Solow residual is probably better over long horizons, where the relative degree of hoarding/effort is does not vary too much.

There is still the question of where the shocks do come from if not from productivity. We can classify shocks into two groups, those that are common to RBC and non-RBC models, and those that are not. Of course within the common group, there are differences in emphasis. Also differences in the channel through which the shocks operate.

common: tech shocks, taste shocks, govt purchases, tax changes, foreign stuff (eg oil).

government purch: different channels: RBC: increase in real interest rate leads to lower leisure. Keynesian story: increased demand for goods and labor moves along rigid nominal wages or prices.

Non-common: animal spirits of investors or consumers; money.

Of all of these, the most controversial is surely money supply. Also, the channel through which money affects output in the Keynesian model is the same as that by which govt purchases affect output, and so "money mattering" is a shorthand phrase that also takes account of the different
channels through which government purchases, etc., might matter.

The question of whether and why "money matters" for output (or any other real variables) is one of the oldest and most honorable in economics.

The classic work in this area is Friedman and Schwartz's Monetary History of the United States. This book essentially takes an event-history approach to the relationship between the money supply and the level of real output in the US, showing that changes in the money supply preceded changes in output -- the most dramatic example being the contraction in the money supply which preceded the Great Depression. This work is extended by Romer and Romer.

What do the RBC folks say about the money/income relationship? Is it further evidence against the "real" part of real business cycle models? Counter argument is reverse causation:

(avg money growth and growth of retail sales (Abel and Bernanke)

<table>
<thead>
<tr>
<th>quarter</th>
<th>money</th>
<th>retail sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>-.72%</td>
<td>-4.38%</td>
</tr>
<tr>
<td>II</td>
<td>1.00</td>
<td>1.53</td>
</tr>
<tr>
<td>III</td>
<td>.52</td>
<td>-1.44</td>
</tr>
<tr>
<td>IV</td>
<td>1.55</td>
<td>7.90</td>
</tr>
</tbody>
</table>

There is a boom in both economic activity and money growth every 4th quarter, but clearly money is not causing output.

There are two stories for this reverse causation:
1) endogenous money multiplier
2) endogenous fed behavior.

The second seems to be the right one for the fourth quarter boom.

Manuelli, in his essay introducing the Prescott/Summers debate takes the position that there are two wings of the RBC school: one which says that the RBC model explains everything, and one which says that the transmission mechanism identified by RBC types is important, but that other things (money, sticky prices, etc.) may also matter. But this seems a cop-out. The RBC model is only interesting if it is the only mechanism for explaining fluctuations.

more general problem of internal vs external consistency -- maybe should apply this to other areas of macro (consumption, eg).

[Note for somewhere (see homework problem as well): King and Rebelo say that allowing for variable capital utilization can greatly reduce the required size of technology shocks.]
Problems

1. Consider a one-period model in which individuals make a labor-leisure decision. Individuals are endowed with one unit of time. Utility is given by:

\[ U = \ln(n) + \frac{c^{1-\sigma}}{1-\sigma} \quad \sigma > 0 \]

where \( n \) is leisure and \( c \) is consumption. People take the wage rate, \( w \), as exogenous. They live for only one period, and consume all of their earnings.

Show how the effect of the wage on labor supply depends on the value of \( \sigma \). For what values of \( \sigma \) will an increase in the wage have a positive effect on labor supply, and for what values will it have a negative effect?

2. Consider an individual who lives for two periods. Her utility function is

\[ U = \ln(c_1) + \ln(c_2) + \alpha \ln(n_1) + \alpha \ln(n_2) \]

Where \( c \) is consumption and \( n \) is leisure. There is no discounting and the interest rate is zero. She is endowed with one unit of time in each period. Wages in both periods are initially \( w_1 = w_2 = w \).

Calculate the short run elasticity of labor with respect to the wage; that is, the increase in period one labor supply in response to an increase in the wage in period one. Now calculate the long run elasticity of labor supply, the response of labor in period one to an increase in wages in both periods.

3. (final exam, 2002) Consider a world in which the only form of output is fruit that falls from trees. The time period is days. The fruit falls from the trees only on odd numbered days. Specifically, on each odd numbered day, one unit of fruit per capita falls. The fruit can be eaten on the day it falls or stored to be eaten at any day in the future, with zero depreciation (there is no integer constraint, that is, people can eat or store fractions of a fruit.)

The population is composed of identical, infinitely lived people with log utility and time discount rates \( \theta > 0 \). At time period 1 (the first period in which this world exists), there is no existing stockpile of fruit.

Solve for the interest rate that holds between odd and even periods and the interest rate that
will hold between even and odd periods.

4. Consider the problem of an individual maximizing lifetime utility.

\[ V = \sum_{t=0}^{\infty} (1 + \theta)^t U(C_t, N_t) \]

s.t. \[ \sum_{t=0}^{\infty} (C_t - w_t(1 - N_t))(1 + r)^t = 0 \]

Assume that \( r = \Theta \). Wages follow the pattern

\[ w_t = w^h \text{ for } t=1,3,5,... \]
\[ w^f \text{ for } t=0,2,4,... \]

where \( w^h > w^f \). Compare the pattern of labor supply of a person with time separable utility

\[ U_t = \ln(C_t) + \ln(N_t) \]

with time non-separable preferences specified as follows:

\[ U_t = \ln(C_t) + \ln((N_t + N_{t-1})/2) \]

5. Assume that the following facts are true: 1) Over the long term, the real wage has risen dramatically. 2) Over the long term, the fraction their time that people spend working has remained constant. 3) Over the long term, the ratio of per capita consumption to the real wage has remained constant.

Assume that the utility function is

\[ U_t = \frac{c_t^\gamma}{1-\sigma} + \beta \frac{n_t^\gamma}{1-\gamma} \]

where \( c \) it consumption, \( n \) is leisure, and the time endowment per period is one.

Based on the assumed facts, what can you conclude about the value of \( \sigma \)?

6. [Core exam, 2001] An individual lives for two periods. There is no uncertainty. She is born and dies with zero assets. She does consumption \( c_1 \) and \( c_2 \), has leisure \( n_1 \) and \( n_2 \), and earns wages \( w_1 \) and \( w_2 \). She has an endowment of one unit of time per period. There is some real interest rate, \( r \), that holds between the two periods, and some time discount rate, \( \Theta \).

The within-period utility function is
\[ U = \frac{c^{1-\sigma}}{1-\sigma} + \frac{n^{1-\sigma}}{1-\sigma} \quad \sigma = 4 \]

We do not observe the real interest rate, the time discount rate, or the wage rate in either period (although these are all known to the individual.)

The following facts are observed

\[ n_1 = n_2 \]
\[ c_2 = 2c_1 \]

A. Calculate the ratio of second period wages to first period wages, \( w_2/w_1 \).

B. Suppose that the individual in part A had been constrained, so that she could not borrow or lend between periods one and two. Further, suppose that the ratio of \( w_2/w_1 \) had been the same as you found in part A. Finally suppose that optimal first period leisure, \( n_1 \), had been one-half. Solve for optimal second period leisure, \( n_2 \).

7. Consider the usual intertemporal optimization problem with consumption and leisure. Variation in consumption and leisure can be caused by one of two things: variation in the interest rate or variation in the real wage. For which of these two causes will consumption and leisure be positively correlated and for which will they be negatively correlated. Explain.

8. (Midterm exam 2001) Consider a person who gets utility from consumption and leisure, as in the standard RBC model. There is one small difference, however: suppose that there is a price of leisure (for example, the alternative to working is going to movies, which requires spending.)

Specifically, let \( q_t \) be the price of leisure in period \( t \). A person who had \( n_t \) units of leisure in period \( t \) would have to spend \( n_t q_t \) dollars on leisure.

Now consider a model where the within-period utility function is

\[ U = \ln(c) + \ln(n) \]

Consider a person who lives for one period and takes \( w \) and \( n \) as exogenous. Find the sign of the derivatives \( dn/dw, dn/dq, dc/dw, \) and \( dc/dq \).
Suppose that price of leisure fell (because people learned to enjoy books rather than movies.) What would happen to output? What would happen to utility?

8.5) [final exam, 2005] Consider a real business cycle model in which the utility that people get from leisure depends on how many other people are having leisure at the same time. Specifically, suppose that utility for an individual is given by

\[ u = \ln(n - \phi \bar{n}) + c \]

Where \( n \) is leisure, \( \bar{n} \) is average leisure, and \( 0 < \phi < 1 \). The capital stock per worker is fixed at \( \bar{k} \). There is no way to save, so consumption is equal to output. The production function per worker is

\[ y = A\bar{k}^\alpha (1 - n)^{1-\alpha} \]

Factors are paid their marginal products. Capital income is earned by people outside the country whose leisure is irrelevant to workers.

A) [5 points] Taking the wage as given, solve for the equilibrium level of leisure.

B) [5 points] Show how you would solve for the derivative of the wage with respect to \( A \). You don’t actually have to find a closed-form solution, because the algebra is tedious. But you should write down the set of equations that you would solve.

C) [5 points] Is the response of wages to a shock to \( A \) greater in the case where \( \phi = 0 \) or when \( \phi > 0 \)? Explain why in terms of the underlying economics.

9. (Core exam, 2003) A woman is born at time zero and will live forever. She is born with zero assets. She can borrow or lend at interest rate \( r > 0 \). She is endowed with one unit of time per period, which she can use for working or leisure. Time is continuous. Her lifetime utility function is

\[ U = \int_0^\infty e^{-\theta t} \left[ \ln(n) + \ln(c) \right] dt \]

where \( \theta \) is the time discount rate, \( n \) is leisure, and \( c \) is consumption. Her wage at time zero, \( w(0) \), is equal to one. Following time zero, her wage grows at rate \( g \).
The relation between $r$, $\theta$, and $g$ is as follows:

$$\theta = g = \frac{r}{2}$$

Solve for her optimal lifetime paths of consumption and leisure. Specifically, solve for $c(0)$ and $n(0)$, and describe mathematically the paths of consumption and leisure after time zero.

Note: You can solve this problem by using a lot of math, but you don't need to. You should be able to do it by simply putting together a bunch of things you already know.

---

10. Consider a model where output is produced by labor alone:

$$y = f(l) \quad f'>0 \quad f''<0$$

There is no capital, and thus no means to transfer output from one period to another. Utility is defined over consumption ($c$) and leisure ($n$), and people are endowed with one unit of time per period.

Taxes are zero in every year except year $s$. In that year, the government will spend some amount $g_s$. It can either collect this amount as a lump sum tax or with a proportional tax at rate $\tau$. Assume that $\tau$ is set so that these two policies generate the same revenue. How do the two policies differ in their effect on the interest rate that holds between periods $s$ and $s+1$?

---

10.5) [final exam, 2005] Consider an economy in which output is produced by labor alone. The production function is (in per capita terms).

$$y_t = z_t (1 - n_t)$$

Where $z$ is a stochastic productivity shock and $n$ is leisure. All of output is paid to workers. There is no way to store output between periods, so output is equal to consumption.

Workers have the following utility function.

$$u_t = \ln(c_t - \gamma c_{t-1}) + \ln(n_t - \alpha n_{t-1})$$

Where $-1 < \gamma < 1$ and $-1 < \alpha < 1$.

[5 points] Discuss the economic interpretation of the parameters $\gamma$ and $\alpha$. What does it mean
economically for these parameters to be positive vs. negative.

[5 points] What should the signs of \( \gamma \) and \( \alpha \) be to produce the largest response of output to the productivity shock \( z \)? You don’t need to do any algebra here or find the exact values. I just want an intuitive explanation.

[5 points] What should the signs of \( \gamma \) and \( \alpha \) be to produce the most persistent effect of a productivity shock on output? As in part B, you don’t need to do any algebra here or find the exact values. I just want an intuitive explanation.

11. (final exam 2001) Consider a worker who has utility for period \( t \)

\[
\ln(c) + \ln(n)
\]

Let each period be just one day. The worker lives from many days, and is trying to choose his labor supply in period \( t \). We assume that the period is small enough relative to the rest of his life that he can take the marginal utility of consumption as constant. Call \( U_c \) the marginal utility of consumption.

Suppose that rather than being able to choose \( n \) continuously (between zero and one), workers are restricted to only three possible choices: \( n=1 \) (no work), \( n=2/3 \) (part time work), or \( n=1/3 \) (full time work).

A. Solve for the worker's labor supply curve. Label any important levels of wages on the curve.

Given this labor supply curve, discuss how changes in labor demand will affect the wage and the level of labor input.

B) Now suppose that part time work is eliminated (but only for the day in question; the rest of the person's life is unchanged.) So the worker has the choice of \( n=1 \) or \( n=1/3 \). Graph the person's labor supply curve in this case, and show how it compares to the case in part A. Is it possible that eliminating part time work can raise the level of output?

12. (midterm exam, 2002) A worker has the following utility function:

\[
U = \begin{cases} 
  c + \ln(n) & \text{if } n < 1 \\
  c + \ln(n) + \gamma & \text{if } n = 1 
\end{cases}
\]
where $\gamma > 1$, $c$ is consumption, and $n$ is leisure. The justification for this utility function is that $\gamma$ measures the extra utility that a person gets from not having to do any work at all (for example, the freedom from having to shave.)

The worker lives for one period. She takes the wage as given. Solve for and sketch out the labor supply curve of the worker. In particular, find the equation that gives the value of $w$ at which there is a discontinuity.

13. Consider an OLG model in which there is capital mobility. There are two countries in the world, with equal populations. There is no population growth, technological progress, or depreciation. The production functions in the two countries are the same: $Y = K^{0.5}L^{0.5}$. In each country, people work in the first period of life only, and consume in both the first and second periods of life. Their utility functions are:

Country 1: $U = (1-\gamma) \ln(c_y) + \gamma \ln(c_o)$

Country 2: $U = (1-\beta) \ln(c_y) + \beta \ln(c_o)$

where $c_y$ is consumption when young, $c_o$ is consumption when old, and $1 > \gamma > \beta > 0$.

A. [10 points] Solve for the steady-state level of GDP per worker in each country.

B. [10 points] Solve for the steady-state level of GNP per worker in each country.

14. [old core exam question] Consider a model in which individuals live for two periods. Their lifetime utility functions are:

$U = \ln(c_1) + \ln(c_2) + \ln(n_1) + \ln(n_2)$

Where $c$ is consumption and $n$ is leisure. Each individual has an endowment of one unit of time per period.

There are two types of individuals. Some individuals earn a wage rate of 1 in the first period and 2 in the second period, while other individuals earn a wage of 2 in the first period and 1 in the second period. Individuals cannot borrow against their future wage income, but they can save between periods at a real interest rate of zero. Individuals are born with zero assets and may not die in debt.

Calculate total earnings (that is, wage multiplied by time spent working) in each period for each of the two types of people. For which group will the variation in earnings between the two periods be higher?
15. Consider an individual who lives for two periods. His utility function is

\[ U = \ln(c_1) + \ln(c_2) + \ln(n_1) + \ln(n_2) \]

Where \( c \) is consumption and \( n \) is leisure. He has an endowment of one unit of time per period. In each period, the wage is equal to one. He can borrow or lend at interest rate \( r \).

Calculate the elasticity of first period saving with respect to the interest rate.

16. An economy which lasts for two periods is composed of identical individuals. In the first period, income per capita is 1. In the second period income per capita will be equal to .5 with probability of one half, and equal to 1.5 with probability one half. There is no capital in this economy, and thus no way that aggregate consumption can differ from aggregate income.

Individuals have utility functions:

\[ U = \ln(c_1) + \ln(c_2) \]

An asset with zero net supply promises to pay \((1+r)\) dollars in period two, in return for a payment of one dollar in period one. Calculate the value of \( r \) for which the asset will have zero excess demand.

17. (final exam, 2001) A person will live for one period. Her utility function is

\[ U = \ln(n) + \ln(c) \]

where \( n \) is leisure and \( c \) is consumption. She is endowed with one unit of time that she can spend working or enjoying leisure. Let \( w \) be the wage per unit of time. There is uncertainty about what the wage will be.

Consider two scenarios: in scenario A, the person has to specify what her consumption will be before she gets to see the value of \( w \). In scenario B, the person has to specify what her leisure will be before she gets to see the value of \( w \).

In which of these scenarios will her utility be higher? Explain why.

You will notice that I haven't given you the usual set of information in this problem. Specifically, I haven't told you about the distribution of \( w \). This is intentional. You can answer the problem with only the information given here.

18. (final exam, 2002) Consider a two-period model in which identical individuals choose leisure and consumption. The within period utility function is
\[ U = \ln(c) + \ln(n) \]

Individuals are endowed with one unit of time per period. The wage in each period is one. The time discount rate is zero. Output cannot be stored from one period to the next.

There is a 50% chance that a law will be passed in the second period which restricts all individuals to work exactly one quarter time. This uncertainty will not be resolved until after period 1 is over.

Solve for the interest rate that holds between periods one and two.

19) [Core Exam, 2005] Consider the following setup. There is one period during which a man may work, consume, and enjoy leisure. He is endowed with one unit of time, which can be spent either working or having leisure. His utility function is

\[ U = c^{1/2} + n, \]

where \( c \) is consumption and \( n \) is leisure. His wage is uncertain. With probability .5 it will be \( w=1 \); with probability .5 it will be \( w=0 \).

Prior to the period in which consumption takes place, and prior to observing what his wage will be, the man has the opportunity to buy insurance against a low wage by selling a share of his labor income. Call the share that he sells \( \alpha \). The firm that buys the labor income agrees to pay a fixed amount \( X \) to the man in the state of the world where \( w=0 \), but nothing in the state of the world in which \( w=1 \). In return, the man will pay the firm some share \( \alpha \) of his labor income in the good state of the world.

There is a perfectly competitive market of buyers who will purchase this labor income. Buyers are risk neutral and earn zero expected profits. The share of his income that the man sells is fully observable by buyers, and the man must commit to sell this share and no more before any transactions take place. The man cannot pre-commit to work a fixed fraction of his time, however.

What is the optimal fraction of his labor income that the man should sell?

Note: the algebra in this question is more messy than usual. In particular, depending on how you solve it, you may end up with a quadratic equation with two roots, only one of which is an optimum. You may leave the solution in the form of this quadratic equation. You do not have to determine which of the roots is an optimum. Alternatively, you can simply write down an implicit equation for \( \alpha \).
20. (Midterm exam, 2002) This question examines the effects of allowing for variable capital utilization in an RBC model.

A firm has production function.

\[ Y_t = (z_t K_t)^\alpha L_t^{1-\alpha} \]

Where \( z_t \) is the rate of capital utilization (in the data, \( z_t \) might be bounded from above by 1, but we ignore this constraint for convenience.) The level of capital at time \( t \) is fixed (in other words, there is no capital market and no rental rate for capital.) The quantity of depreciation is \( \delta_t K_t \). The price of the output good is normalized to one. The firm takes as exogenous the wage, \( w_t \). The firm seeks to maximize output less depreciation and less wages, \( w_t L_t \).

Let the rate of depreciation \( \delta_t \) be a function of \( z_t \). Specifically,

\[ \delta_t = \beta z_t^2 \]

Where \( \beta > 0 \).

A. Consider the case where \( z \) is fixed at \( z=1 \). Solve for the elasticity of demand for labor, that is, \( \frac{d \ln(L)}{d \ln(w)} \).

B. Now allow the firm to choose \( z \) optimally. Solve for the elasticity of demand for labor (this involves some tedious algebra). Show how it compares to the case in part A.

21) [Core Exam, 2005] Consider a world in which people live two periods. Each individual has one child and one parent. All individuals in a generation are identical.

In the first period of life, people are children living in their parent’s house. During this period, individuals make no decisions. Their parents pay for their consumption.

In the second period of life, individuals are endowed with one unit of time, which they split between work and leisure. The wage is equal to 1 per unit time. Their utility function of someone who is in the second period of life in period \( t \) is

\[ U = \ln(n_{2,t}) + \ln(c_{1,t-1} + c_{2,t}) \]

where \( n_{2,t} \) is his leisure, \( c_{2,t} \) is his consumption during period 2 of his life, and \( c_{1,t-1} \) is the consumption that he did when he was young.

Parents face the constraint that they must provide their child with the same level of consumption that they have, that is \( c_{1,t} = c_{2,t} \).
Analyze the dynamics of second period consumption. In particular, find the steady state level of consumption, and show how consumption varies over time if it starts at some level other than the steady state.

Explain briefly (roughly 3-4 paragraphs) what the particular characteristic(s) of the utility function lead to these consumption dynamics and why they do so. Also discuss whether this characteristic of the utility function is reasonable, and what an alternative would be. How would such an alternative formulation of the utility function affect the steady state and dynamics of the model?