Money and Search - The Kiyotaki-Wright Model

Econ 208

Lecture 14

March 20, 2007
Problem with the OLG model - can account for alternative stores of value but not for “rate of return dominance”

Search theory - tries to provide an explicit account of the liquidity yield of money

A more convincing model of the “double coincidence” problem

Wicksell’s triangle

Will describe the Kiyotaki-Wright (1993) model of fiat money
Basic Assumptions

- Continuum of people on $[0, 1]$
- Continuum of goods $i \in [0, 1]$
- Each good is storable and indivisible (available in quantity 0 or 1)
- Fiat money - also storable and indivisible (available in quantity 0 or 1)
- Preferences - each person can consume only a fraction $x$ of goods, and each good can be consumed by a fraction $x$ of people.
- $r$ rate of time preference
- $U$ the utility of consuming one of your “consumption goods”
- The fraction $M$ of people are initially endowed with money
- The fraction $1 - M$ are initially endowed with one real commodity
More Assumptions

Production

- Once you have consumed you enter the production mode in which a production opportunity arrives with Poisson arrival rate $\alpha$
- Each opportunity results in 1 unit of a good the producer can’t consume

Exchange

- After production you enter a market where you look for a trading partner.
- There you meet other people with a Poisson arrival rate $\beta$
- Because of anonymity a *quid pro quo* is required
- There is a transaction cost $\varepsilon \in (0, U)$ from receiving something other than money from someone
- Exchange will take place if and only if it is mutually beneficial, at an exchange rate of unity
More of the setup

- At any point of time all “traders” (people in the market) are either “goods traders” (holding one unit of money) or “money traders”
- $\mu$ is the fraction of traders who are money traders (buyers)
- $1 - \mu$ the fraction who are goods traders (sellers)
- Agents must adopt a trading strategy - when to accept various goods and when to accept money
- We look for a symmetric stationary Nash equilibrium in trading strategies
Need to characterize the traders’ best-response functions

It is *always* best to accept a good you can consume (otherwise you just delay)

It is *never* best to accept a commodity you can’t consume (because of symmetry and $\varepsilon$)

Therefore $x$ is the probability a random goods traders will accept the good of another random goods trader (a “single coincidence”), and $x^2$ is the probability that each will accept the other’s good

The more difficult choice is whether to accept money - will depend on the probability $\Pi$ that others will do the same
Let $V_j$ be the value (expected present value of utility) of a person in state $j \in \{0, 1, m\}$ where the states are defined by what the person is holding:

- $j = 0$ producer
- $j = 1$ goods trader
- $j = m$ money trader

The $V$'s satisfy these Bellman equations:

\[
\begin{align*}
    rV_0 &= \alpha (V_1 - V_0) \\
    rV_1 &= \beta (1 - \mu) x^2 (U - \varepsilon + V_0 - V_1) + \beta \mu \times \max \{\pi (V_m - V_1)\} \\
    rV_m &= \beta (1 - \mu) x \Pi (U - \varepsilon + V_0 - V_m)
\end{align*}
\]

- You will accept money if $V_m > V_1$, not accept if $V_m < V_1$
- Therefore you accept if $\Pi > x$, not if $\Pi < x$.
- Therefore there are multiple equilibria (see diagram of best-response function)
Consider the limiting case of instant production \((\alpha = \infty)\). Then \(\mu = M\) and \(V_0 = V_1\), so the Bellman equations reduce to:

\[
\begin{align*}
    rV_1 &= \beta (1 - M) x^2 (U - \varepsilon) + \beta M x \max \{ \pi (V_m - V_1) \} \\
    rV_m &= \beta (1 - M) x \Pi (U - \varepsilon + V_1 - V_m)
\end{align*}
\]

so the values of the two states in the three equilibria are:

<table>
<thead>
<tr>
<th>Equilibrium</th>
<th>(V_m)</th>
<th>(V_0)</th>
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</thead>
<tbody>
<tr>
<td>(\Pi = 0)</td>
<td>0</td>
<td>(\beta (1 - M) x^2 (U - \varepsilon) / r)</td>
</tr>
<tr>
<td>(\Pi = x)</td>
<td>(\beta (1 - M) x^2 (U - \varepsilon) / r)</td>
<td>(\beta (1 - M) x^2 (U - \varepsilon) / r)</td>
</tr>
<tr>
<td>(\Pi = 1)</td>
<td>&gt; (\beta (1 - M) x^2 (U - \varepsilon) / r)</td>
<td>&gt; (\beta (1 - M) x^2 (U - \varepsilon) / r)</td>
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So the monetary equilibrium \((\Pi = 1)\) Pareto-dominates the others.
Commentary

- The model can account for rate-of-return dominance (there can be a small cost of holding money)
- But many problems
  - Indivisibility of money? (has been relaxed, at a cost)
  - No institutional structure