Problems with the Taylor Model

1. Can’t specify AS independently of AD
2. In reality, there are heterogeneous price-change frequencies
   1. One response - state dependent pricing (S-s policies)
      → the Caplin-Spulber result of policy ineffectiveness
   2. Another response - random time-dependent pricing
      → rationalization - fixed decision costs

The Calvo model deals with both of these problems, by adopting the second strategy - random time-dependent pricing
The Calvo Model

Basic setup

Taken from Calvo (1983). See King (2000) for a more readable account. Same conceptual framework of symmetric imperfectly competitive sellers.

Notation:
- $\beta$: discount factor
- $\omega$: probability you won’t change your price this week
- $x_t$: log of price set at $t$ (the “contract” price at $t$)
- $p_t^*$: log of “ideal” price at $t$
- $p_t$: price index at $t$

Assumption: $x_t$ chosen to maximize expected present value of profits
Solving for the optimal contract price

\[ x_t = \arg\min E_t \sum_{0}^{\infty} \beta^j \omega^j (x_t - p_{t+j}^*)^2 \]  \hspace{1cm} \text{(log-linear approx to EPV max)}

\[ E_t \sum_{0}^{\infty} \beta^j \omega^j 2(x_t - p_{t+j}^*) = 0 \]  \hspace{1cm} \text{(First-order condition)}

\[ (1 - \beta \omega)^{-1} x_t = E_t \sum_{0}^{\infty} \beta^j \omega^j p_{t+j}^* \]

\[ (1 - \beta \omega)^{-1} x_t = p_t^* + \beta \omega E_t \sum_{0}^{\infty} \beta^j \omega^j p_{t+1+j}^* \]

\[ (1 - \beta \omega)^{-1} x_t = p_t^* + \beta \omega (1 - \beta \omega)^{-1} E_t x_{t+1} \]

\[ x_t = (1 - \beta \omega) p_t^* + \beta \omega E_t x_{t+1} \]
Solving for inflation

\[ p_t = \omega p_{t-1} + (1 - \omega) x_t \]  
(The price index)

\[ (1 - \omega) p_t + \omega p_t = \omega p_{t-1} + (1 - \omega) x_t \]

\[ \pi_t = \frac{1 - \omega}{\omega} (x_t - p_t) \]

\[ x_t = (1 - \beta \omega) p^*_t + \beta \omega E_t x_{t+1} \]  
(Optimal contract price)

\[ x_t - p_t = (1 - \beta \omega) \phi y_t + \beta \omega E_t (x_{t+1} - p_{t+1} + \pi_{t+1}) \]

\[ \pi_t = \frac{1 - \omega}{\omega} \left[ (1 - \beta \omega) \phi y_t + \beta \omega E_t \left( \frac{1}{1 - \omega} \pi_{t+1} \right) \right] \]

\[ \pi_t = \alpha y_t + \beta E_t \pi_{t+1} \]  
(New Keynesian Phillips Curve)

where

\[ \alpha = \left[ \frac{(1 - \omega) (1 - \beta \omega)}{\omega} \right] \phi > 0 \]
The New Keynesian Phillips Curve

\[ \pi_t = \alpha y_t + \beta E_t \pi_{t+1} \]  
(New Keynesian Phillips Curve)

Different from the new classical curve \( \pi_t = \phi E_{t-1} y_t + E_{t-1} \pi_t \)

1. Current output instead of lagged expectation of current output
2. Coefficient of \( y_t \) is just proportional the elasticity \( \phi \) of marginal cost
3. Expected future inflation enters instead of lagged expectation of current inflation
4. The coefficient \( \beta \) on expected inflation is strictly less than unity (but not by much)

So you don’t get the policy ineffectiveness proposition. Instead:

\[ \pi_t = \alpha \sum_{j=0}^{\infty} \beta^j E_t y_{t+j} \]

so \( y_t = y > 0 \) is possible with steady inflation:

\[ \pi = \alpha \frac{1}{1 - \beta} y \]

but the tradeoff is very steep
The New Keynesian Phillips Curve
Alternative derivation (Rotemberg, 1987)

Suppose that all firms change prices each period, but subject to a cost equal to

\[ c \cdot (p_t - p_{t-1})^2 \]

So all prices will be the same, but they will change gradually. Maximization of expected present value of profit now implies that \( p_t \) will be chosen to:

\[
\min E_t \sum_0^\infty \left[ (p_{t+j} - p_{t+j}^*)^2 + c \cdot (p_{t+j} - p_{t+j-1})^2 \right]
\]

(log-linear approx to EPV max)

\[ 2 (p_t - p_t^*) + 2c (p_t - p_{t-1}) - \beta 2c \left( E_t p_{t+1} - p_{t} \right) = 0 \]

(First-order condition)

\[ (1/c) (p_t - p_t^*) + \pi_t - \beta E_t \pi_{t+1} = 0 \]

\[ \pi_t = (1/c) \phi y_t + \beta E_t \pi_{t+1} \]
The New Keynesian Phillips Curve

Problems

1. No inflation inertia. If you are having a big inflation, you can just cut announce $y_{t+j} = 0$ for all $j$ and then:

$$\pi_t = \alpha \sum_{0}^{\infty} \beta^j E_t y_{t+j} = 0$$

2. The model predicts that $\Delta \pi_t$ should be counter-cyclical:

$$-\pi_t = -\alpha y_t - \beta E_t \pi_{t+1}$$

$$E_t (\pi_{t+1} - \pi_t) = -\alpha y_t + (1 - \beta) E_t \pi_{t+1}$$

Responses to these second problem:

1. Instead of $\alpha y_t$ we should really have something proportional to log marginal cost, which is itself countercyclical (?) Gali-Gertler, 1999

2. Need to re-introduce **lagged** inflation on the RHS of the Phillips Curve
The New Keynesian Phillips Curve
Backward-looking elements

An extreme case of introducing lagged inflation would be:

$$\pi_t = \alpha y_t + \pi_{t-1}$$

Then there would be inflation inertia because you need a recession to reduce $\pi_t$

$$\Delta \pi_t = \alpha y_t$$

and this would again make $\Delta \pi_t$ pro-cyclical.

Alternatively you could retain the NKPC but replace rational expectations with adaptive:

$$\pi_t = \alpha y_t + \beta E_t \pi_{t+1}$$

$$\pi_t = \alpha y_t + \beta \sum_{i=1}^{\infty} \gamma_i \pi_{t-i}$$

Or you can assume a “hybrid” model:

$$\pi_t = \alpha y_t + \beta_f E_t \pi_{t+1} + \beta_b \pi_{t-1}$$

When a hybrid model is estimated it almost always produces $\beta_b > 0$. 