Focus on long run aspects

- what determines inflation and the price level?
- how does money affect real economic activity?

Central issue is how to integrate money with the real side of the economy

To do this we need to introduce some sort of friction

Otherwise we are left with fundamental questions

- Why do we use money?
- Why do people accept money?
- Why do people hold money?
- Why does money have value in equilibrium?
Money
The Classical analysis

The quantity theory of money

\[ MV = PT \quad \text{or} \quad M/P = L(y, r) \]

The classical dichotomy

- a dichotomy between the real side of the economy and the monetary side
- it is the proposition that
  1. real variables (including relative prices) are determined independently of monetary factors, and
  2. monetary factors along with real variables determine the price level
- this dichotomy is invoked by the quantity theory
The neutrality of money

- this is the proposition that changes in the quantity of money do not affect real variables.
- it is implied by, but does not imply, the classical dichotomy
- mainstream macro supports the neutrality proposition as a long-run proposition, but not short-run

The superneutrality of money

- this is the proposition that changes in the rate of growth of the quantity of money do not affect real variables.
- superneutrality is controversial even as a long run proposition
A simple (some would say too simple) way to integrate money into the analysis of real variables

Reference - Blanchard-Fischer, ch.4 or my notes on Josh’s web site

Basic idea for getting money into the analysis - utility depends not just on consumption but also on the holding of real money balances

\[ u(c, m) \]

- rationalization - Patinkin’s stochastic payments process

Otherwise the Sidrauski model is the same as the Ramsey model
The Sidrauski Model
Conceptual Framework

Population grows at the fixed rate \( n \)
3 types of agent

1. Firms
2. Households
3. Government

3 tradeable objects

1. Output - used for consumption or capital - depreciates at rate \( \delta \)
2. Labor
3. Money
Households

- rent $K$ and $L$ to firms
- buy $Y$ from firms
- trade $K$ and $M$ with each other

Firms

- produce according to $Y = F(K, L) = C + I$

Government

- Make lump-sum transfers to households, financed by printing money
  
  $\dot{M} = TR$

  where $TR$ could be negative
At each date $t$

- $M(t), K(t)$ are historically predetermined
- people have perfect foresight about transfers and all future prices
- the auctioneer determines $(P(t), W(t), R(t))$ to equate supply and demand for output, labor and money
The Sidrauski Model

Firm behavior

Each period, firms choose $K, L$ to

$$\max F(K, L) - (R/P)K - (W/P)L$$

so we get the usual first-order condition:

$$F_1(K, L) = R/P, \quad F_2(K, L) = W/P$$

which implies, in intensive form

$$f'(k) = r + \delta$$
$$f(k) - k f'(k) = w$$
$$f(k) = c + i$$

where $k = K/L$, $r = (R/P) - \delta$, $w = W/P$, $c = C/L$ and $i = I/L$.
Given \( tr = TR / (PL) \) and \( \pi = \dot{P} / P \), households choose time paths for \( c \), \( m = M / (PL) \) and \( a = m + k \), to

\[
\max \int_0^\infty e^{-(\rho - n)t} u \left( c(t), m(t) \right) dt
\]

subj to \( \dot{a} = w + tr + (r - n) a - c - (r + \pi) m \)

This is a straightforward optimal control problem with one state variable \( a \) and two control variables \( c \) and \( m \).

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\(^1\)As an exercise, derive this constraint from the def’n of \( a \) and:

\[
\dot{M} = WL + RK + TR - PC - PI, \ \dot{K} = I - \delta K
\]
The Hamiltonian is

\[ H = u(c, m) + \lambda (w + tr + (r - n) a - c - (r + \pi) m) \]

Maximizing \( H \) with respect to the two controls yields the FOCs:

\[
\begin{align*}
    u_c(c, m) &= \lambda \\
    u_m(c, m) &= (r + \pi) \lambda
\end{align*}
\]

the Euler equation:

\[
\dot{\lambda} = (\rho - n) \lambda - \partial H / \partial a = (\rho - r) \lambda
\]

and the Transversality condition:

\[
e^{-(\rho - n)t} \lambda a \to 0 \text{ as } t \to \infty
\]
The Sidrauski Model

Equilibrium

Assume constant money growth: $\dot{M}(t) = TR(t) = \mu M(t)$ for all $t$.
An equilibrium wrt $(K_0, M_0, \mu)$ is $\{c, m, \lambda, r, \pi, w, k, P\}_{t=0}^{t=\infty}$ such that

$$k(0) = K_0, \ m(0) = M_0 / P(0), \ \lim_{t \to \infty} e^{-(\rho - n) t} \lambda a = 0$$

and for all $t$:

$$\lambda = u_c(c, m)$$
$$\lambda = u_m(c, m)$$
$$\lambda = (\rho - r) \lambda$$
$$w = f(k) - k f'(k)$$
$$r = f'(k) - \delta$$
$$\dot{k} = f(k) - (n + \delta) k - c$$
$$\dot{m} = (\mu - n - \pi) m$$
$$\dot{P} = \pi P$$