Chapter 7
Capital, Innovation, and Growth Accounting

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1 Introduction

Neoclassical theory and AK theory focus on capital accumulation, whereas the product variety and Schumpeterian theories focus on innovations that raise productivity. One way to judge the competing theories is to ask how much of growth is attributable to the accumulation of physical and human capital, and how much is the result of productivity growth. This can be done using “growth accounting”, a method first invented by Solow (1958). This chapter shows how growth accounting works.

Economists who have conducted growth accounting exercises in many economies (for example, Jorgensen 1995) have concluded that a lot of economic growth is accounted for by capital accumulation. These findings raise a number of issues that we also deal with in this chapter. For one thing, the results of growth accounting are very sensitive to the way capital is measured. We discuss below some cases in which there is reason to believe that capital is systematically mismeasured. One of these cases concerns the claim by Alwyn Young (1995) that most of the extraordinary growth performance of Singapore, Hong Kong, Taiwan and South Korea can be explained by factor accumulation, not technological progress. Hsieh (2002) argues that these results are no longer true once one corrects for the overestimates of capital accumulation in the data.

Another issue raised by growth accounting has to do with the difference between accounting relationships and causal relationships. We will show in this chapter that even though there is evidence that somewhere between 30 and 70 percent of the growth of output per worker in OECD countries can be “accounted for” by capital accumulation, nevertheless these results are consistent with the neoclassical model which implies that in the long run all of the growth in output per worker is caused by technological progress.

In this chapter we also show how capital can be introduced into the Schumpeterian model that we are using as our main workhorse model throughout the book. The result is a hybrid
model in which capital accumulation takes place as in the neoclassical model but productivity growth arises endogenously as in the Schumpeterian model. The hybrid model is consistent with the empirical evidence on growth accounting, as is the neoclassical model. But the causal explanation that it provides for economic growth is quite different from that of the neoclassical model.

2 Measuring the growth of total factor productivity

When people mention productivity, often what they are referring to is “labor productivity”, which is output per worker: \( y = Y/L \). But this particular measure of productivity confounds the effects of capital accumulation and technological progress, both of which can raise output per worker. To see this, suppose that output depends on capital, labor and a productivity parameter \( B \) according to the familiar Cobb-Douglas aggregate production function:

\[
Y = BK^\alpha L^{1-\alpha}
\]

Then dividing both sides by \( L \) we see that output per worker equals:

\[
y = Bk^\alpha
\]

where \( k = K/L \) is the capital stock per worker. So according to (2), labor productivity \( y \) depends positively on the productivity parameter \( B \) but also on the capital stock per worker \( k \).

A better measure of productivity, which separates technological progress from capital accumulation, is the parameter \( B \). This parameter tells us not just how productive labor is, but how productively the economy uses all the factors of production. For this reason, \( B \) is called “total factor productivity”, or just “TFP”.

Our measure of economic growth is the growth rate \( G \) of output per person. Under the simplifying assumption that the population and labor force grow at the same rate, \( G \) is also
the growth rate of output per worker. So from (2) we can express the growth rate as:

$$G = \dot{B}/B + \alpha \dot{k}/k$$  \hfill (3)

According to (3), the growth rate is the sum of two components: the rate of TFP growth ($\dot{B}/B$) and the “capital deepening” component ($\alpha \dot{k}/k$). The first one measures the direct effect of technological progress, and the second measures the effect of capital accumulation. The purpose of growth accounting is to determine the relative size of these two components.

If all of the variables in equation (3) could be observed directly then growth accounting would be very simple. However, this is not the case. For almost all countries we have time-series data on output, capital and labor, which allow us to observe $G$ and $\dot{k}/k$, but there are no direct measures of $B$ and $\alpha$. Growth accounting deals with this problem in two steps. The first step is to estimate $\alpha$ using data on factor prices and the second step is to estimate TFP growth ($\dot{B}/B$) using a “residual” method. These two steps work as follows.

First we must make the assumption that the market for capital is perfectly competitive. Under that assumption, the rental price of capital $R$ should equal the marginal product of capital. Differentiating the right-hand side of (1) to compute the marginal product of capital we then get:2

$$R = \alpha Y/K,$$

which we can rewrite as:

$$\alpha = RK/Y.$$

That is, $\alpha$ equals the share of capital income (the price $R$ times the quantity $K$) in national income ($Y$). This share can be computed directly observed data once we observe the factor price $R$.

To conduct the second step of growth accounting we just rewrite the growth equation (3) as:

$$\dot{B}/B = G - \alpha \dot{k}/k$$

which says that the rate of TFP growth ($\dot{B}/B$) is the residual left over after we subtract

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1 Taking natural logs of both sides of (2) we get:

$$\ln y = \ln B + \alpha \ln k$$

Differentiating both sides with respect to time we get:

$$\dot{y}/y = \dot{B}/B + \alpha \dot{k}/k$$

which is the same as (3) because $G = \dot{y}/y$ by definition.

2 That is, $R = \partial Y/\partial K = \alpha BK^{\alpha-1}L^{1-\alpha} = \alpha BK^{\alpha}L^{1-\alpha}/K = \alpha Y/K$. 

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the capital deepening term from the observed growth rate $G$. Once we have estimated $\alpha$ using factor prices we can measure everything on the right hand side. This measure of TFP growth is known as the “Solow residual”.

2.1 Empirical results

From the national accounts it appears that wage and salaries account for about 70 percent of national income in the United States. In other countries the number is roughly the same. So to a first-order approximation the share of capital is about 0.3, and to get a rough estimate of TFP growth we can set $\alpha$ equal to 0.3. Using this value of $\alpha$ and measures of capital stocks constructed from the Penn World Tables\(^3\), we can break down the average growth rate from 1960 to 2000 of all OECD countries. The results are shown in Table 1 below. The first column is the average growth rate $G$ of output per worker over this 40 year period. The second column shows the corresponding TFP growth rate estimated over that period and the third column is the other capital deepening component of growth. The fourth and fifth columns indicate what percentage of growth is accounted for by TFP growth and capital deepening respectively. As this table indicates, TFP growth accounts for about two thirds of economic growth in OECD countries while capital deepening accounts for one third.

Economists such as Jorgensen (1995) have conducted more detailed and disaggregated growth accounting exercises on a number of OECD countries, in which they estimate the contribution of human as well as physical capital. They tend to come up with a smaller contribution of TFP growth and a correspondingly larger contribution of capital deepening (both physical and human capital deepening) than indicated in Table 1. In the United States, for example, over the period from 1948 to 1986, Jorgensen and Fraumeni (1992) estimate a TFP growth rate of 0.50%, which is about 30% of the average growth rate of output per hour of labor input instead of the roughly 58% reported for the United States in Table 1.\(^4\)

TABLE 1 HERE

The main reason why these disaggregated estimates produce a smaller contribution of TFP growth than reported in Table 1 is that the residual constructed in the disaggregated estimates comes from subtracting not only a physical capital deepening component but also

\(^3\)We thank by Diego Comin of NYU for his help in compiling the capital stock estimates underlying this table.

\(^4\)Their table 5 indicates that on average output grew at a 2.93% rate and labor input (hours times quality) grew at a 2.20% rate. It also indicates that 58.1% of the contribution of labor input came from hours, implying an average growth rate in hours of (0.581 · 2.20 =) 1.28% and an average growth rate in output per hour worked of (2.93 - 1.28 =) 1.65%. Their estimate of the residual was 0.50%, which is 30.3% of the growth rate of output per hour worked.
a human capital deepening component. Since the middle of the 20th Century all OECD
countries have experienced a large increase in the level of educational attainment of the
average worker, that is, a large increase in human capital per person. When the contribution
of this human capital deepening is also subtracted we are clearly going to be left with a
smaller residual than if we just subtract the contribution of physical capital deepening. But
whichever way we compute TFP growth it seems that capital accumulation and technological
progress each account for a substantial share of productivity growth - somewhere between
30 and 70 percent each depending on the details of the estimation.

3 Some problems with growth accounting

3.1 Problems in measuring capital, and the tyranny of numbers

One problem with growth accounting is that technological progress is often embodied in new
capital goods, which makes it hard to separate the influence of capital accumulation from
the influence of innovation. When output rises is it because we have employed more capital
goods, or because we have employed better ones? Economists such as Gordon (1990) and
Violante and Cummins (2002) have shown that the relative price of capital goods has fallen
dramatically for many decades. In many cases this is not because we are able to produce
more units of the same capital goods with any given factor inputs but because we are able
to produce a higher quality of capital goods than before, so that the price of a “quality
adjusted” unit of capital has fallen. For example, it costs about the same as ten years ago to
produce one laptop computer, but you get much more computer for that price than you did
ten years ago. But by how much has the real price fallen? That is a very difficult question
to answer and national income accountants, having being trained to distrust any subjective
manipulation of the data, almost certainly do too little adjusting.

To some extent this problem affects not so much the aggregate productivity numbers as
how that productivity is allocated across sectors. Griliches (1994) has argued, for example,
that the aircraft industry, which conducts a lot of R&D, has exhibited relatively little TFP
growth while the airline industry, which does almost no R&D, has exhibited a lot of TFP
growth. If we were properly to adjust for the greatly improved quality of modern aircraft,
which fly more safely, more quietly, using less fuel and causing less pollution than before,
then we would see that the aircraft industry was really much more productive than the TFP
numbers indicate. But at the same time we would see that productivity has not really grown
so much in the airline industry, where we have been underestimating the increase in their
quality-adjusted input of aircraft. More generally, making the proper quality adjustment
would raise our estimate of TFP growth in upstream industries but lower it in downstream industries. In aggregate, however, these two effects tend to wash out.

A bigger measurement problem for aggregate TFP occurs when a country’s national accounts systematically overestimate the increase in capital taking place each year. As Pritchett (2000) has argued this happens in many countries because of government inefficiency and corruption. Funds are appropriated for the stated purpose of building public works, and the amount is recorded as having all been spent on investment in (public) physical capital. But in fact much of it gets diverted into the pockets of politicians, bureaucrats and their friends, instead of being spent on capital. Since we do not have reliable estimates of what fraction was really spent on capital and what fraction was diverted, we do not really know how much capital accumulation took place. We just know that it was less than reported. As a result it is hard to know what to make of TFP numbers in many countries, especially those with high corruption rates.

A similar problem is reported by Hsieh (2002), who has challenged Alwyn Young’s (1995) claim that the Eastern “Tigers” (Singapore, Hong Kong, Taiwan and South Korea) accomplished most of their remarkable growth performance through capital accumulation and the improved efficiency of resource allocation, not through technological progress. Hsieh argues that this finding does not stand up when we take into account some serious over reporting of the growth in capital in these countries.

According to Young’s estimates, in Hong Kong, GDP per capita grew by 5.7 percent a year over 1966–92. Over 1966–90, Singapore’s GDP per capita grew at 6.8 percent a year. South Korea’s also at 6.8 percent, and Taiwan’s at 6.7 percent. Growth in GDP per worker was between one and two percentage points less, reflecting large increases in labor force participation. With just this simple adjustment, the growth rates begin to look less impressive, but are still very high by the standards of other developing countries.

Young adjusts for changes in the size and mix of the labor force, including improvements in the educational attainment of workers, to arrive at estimates of the Solow residual. For the same time periods as before, he finds that TFP growth rates were 2.3 percent a year for Hong Kong, 0.2 percent for Singapore, 1.7 percent for South Korea, and 2.1 percent for Taiwan. He argues that these figures are not exceptional by the standards of the OECD or several large developing countries.

Hsieh argues, however, that there is clearly a discrepancy between these numbers and observed factor prices, especially in Singapore. His estimates of the rate of return to capital, drawn from observed rates of returns on various financial instruments, are roughly constant over the period from the early 1960s through 1990, even though the capital stock rose on 2.8% per year faster than GDP. As we saw in the neoclassical model, technological progress
is needed in order to prevent diminishing marginal productivity from reducing the rate of return to capital when such dramatic capital deepening is taking place. The fact that the rate of return has not fallen then seems to contradict Young’s finding of negligible TFP growth. The obvious explanation for this apparent contradiction, Hsieh suggests, is that the government statistics used in growth accounting have systematically overstated the growth in the capital stock. Hsieh argues that this is particularly likely in the case of owner-occupied housing in Singapore.

Hsieh also argues that instead of estimating TFP growth using the Solow residual method we should use the “dual” method, which consists of estimating the increase in TFP by a weighted average of the increase in factor prices. That is, if there were no TFP growth then the marginal products of labor and capital could not both rise at the same time. Instead, either the marginal product of labor could rise while the marginal product of capital falls, which would happen if the capital labor ratio \( k \) were to rise; or the reverse could take place if \( k \) were to fall. Using this fact one can estimate TFP as the increase in total factor income per worker that would have come about if factor prices had changed as they did but there had been no change in \( k \). By this method he finds that in two out of the four “Tiger” cases TFP growth was approximately the same as when computed by the Solow residual, but that in the cases of Taiwan and Singapore the dual method produces substantially higher estimates. In the case of Singapore he estimates annual TFP growth of 2.2 percent per year using the dual method versus 0.2 percent per year using the Solow Residual.

3.2 Accounting versus causation

When interpreting the results of growth accounting, it is important to keep in mind that an accounting relationship is not the same thing as a causal relationship. Even though capital deepening might account for as much as 70 percent of the observed growth of output per worker in some OECD countries it might still be that all of the growth is caused by technological progress. Consider for example the case in which the aggregate production function is:

\[
Y = A^{1-\alpha} L^{1-\alpha} K^\alpha,
\]

as in the neoclassical model, where technological progress is exogenous.\(^5\) Here \( A \) measures the technical efficiency of labor.

Comparing this to equation (1) above we see that it implies total factor productivity

\(^5\) And also, as we shall see later in this chapter, in the Schumpeterian framework once capital has been introduced.
equal to:

\[ B = A^{1-\alpha} \]

which implies a rate of TFP growth equal to \( 1 - \alpha \) times the rate of labor-augmenting technological progress

\[ \dot{B}/B = (1 - \alpha) \dot{A}/A \]

Now, as we have seen, in the long run the neoclassical model implies that the growth rate of output per worker in the long-run will be the rate of labor-augmenting technological progress \( \dot{A}/A \):

\[ \dot{A}/A = \dot{y}/y \]

In that sense, long-run economic growth is caused entirely by technological progress in the neoclassical model, and yet the model is consistent with the decomposition reported in Table 1 above, because it says that the rate of TFP growth is:

\[ \dot{B}/B = (1 - \alpha) \dot{y}/y. \]

Given the evidence that \( \alpha \) is about 0.3 this last equation implies that TFP growth is about 70 percent of the rate of economic growth, which is just what the evidence in Table 1 reports.

Of course once take into account the accumulation of human as well as physical capital then the estimated rate of TFP growth falls to about 30 percent of economic growth. But that is just what we would get from the above model if we interpreted \( K \) not as physical capital but as a broad aggregate that also includes human capital, in which case \( \alpha \) should be interpreted not as the share of physical capital in national income but the share of all capital in national income. Simple calculations such as the one reported by Mankiw, Romer and Weil (1992) suggest that this share ought to be about two thirds of national income, in which case the above models would again be consistent with the growth accounting evidence since it would imply a rate of TFP growth of about one third the rate of economic growth, even though again the model would imply that in the long run the cause of economic growth is entirely technological progress.

To see what is going on here, recall that the capital deepening component of growth accounting measures the growth rate that would have been observed if the capital-labor ratio had grown at its observed rate but there had been no technological progress. The problem is that if there had been no technological progress then the capital labor ratio would not have grown as much. For example, in the neoclassical model we saw that technological progress is needed in order to prevent diminishing returns from eventually choking off all growth in the capital labor ratio. In that sense technological progress is the underlying cause
of both the components of economic growth - not just of TFP growth but also of capital deepening. What we really want to know in order to understand and possibly control the growth process is not how much economic growth we would get under the implausible scenario of no technological progress and continual capital deepening but rather how much economic growth we would get if we were to encourage more saving, or more R&D, or more education, or more competition, etc. These causal questions can only be answered by constructing and testing economic theories. All growth accounting can do is help us to organize the facts to be explained by these theories.

4 Capital accumulation and innovation

In this section we develop a hybrid Neoclassical/Schumpeterian model that includes both endogenous capital accumulation and endogenous technological progress in one model. As we shall see, it provides a causal explanation of long-run economic growth that is the same as that of the simpler Schumpeterian model without capital, except that there is now an additional explanatory variable that can affect the growth rate, namely the saving rate. The model provides a more complete explanation of growth-accounting results than does the neoclassical model because it endogenizes both of the forces underlying growth, whereas the neoclassical model endogenizes only one of them.

4.1 A simple model

Consider the following variant of the multi-sector Schumpeterian model laid out in Chapter 5 above. There are three kinds of goods in the economy: a final good, a constant measure of specialized intermediate products, and labor. But now we assume that the final good is storable, in the form of capital, and the intermediate goods are produced with capital. In other words, we can interpret the intermediate products as the services of specialized capital goods, and innovations in this model will be quality improvements in these specialized capital goods.

4.1.1 Production and capital accumulation

There is a constant population of \( L \) individuals, each endowed with one unit of skilled labor that she supplies inelastically. The final good is produced competitively using the intermediate inputs and labor, according to a production function like the one in Chapter 5:

\[
Y_t = \int_0^1 A_t L^{1-\alpha} x_t^{\alpha} di, \quad 0 < \alpha < 1, \tag{4}
\]
where each \( x_{it} \) is the flow of intermediate input \( i \) used at \( t \), and \( A_{it} \) is a productivity variable that measures the quality of the input. The final good is used in turn for consumption, research, and investment in physical capital. For simplicity, let us normalize total labor supply \( L \) at one.

Intermediate inputs are all produced using capital according to the production function

\[
x_{it} = \frac{K_{it}}{A_{it}}
\]

where \( K_{it} \) is the input in capital in sector \( i \). Division by \( A_{it} \) reflects the fact that successive vintages of intermediate input \( i \) are produced by increasingly capital-intensive techniques.

As in the neoclassical model we assume that there is a fixed saving rate \( s \) and a fixed depreciation rate \( \delta \). We also assume that time is continuous. So the rate of increase in the capital stock \( K_t \) will be exactly the same as in the neoclassical model:

\[
\dot{K}_t = sY_t - \delta K_t.
\] (5)

### 4.1.2 Profit maximization by local monopolists

In each intermediate sector things look much the same as they did in the multi-sectoral model of chapter 5 except that, because the intermediate product uses capital as its input, the marginal cost of each local monopolist will depend on the rental rate \( R_t \) on capital. Specifically, since it takes \( A_{it} \) units of capital to produce each unit of the intermediate product, the marginal cost will be \( R_tA_{it} \).

Later on we will make use of the fact that the rental rate is the rate of interest plus the rate of depreciation:

\[
R_t = r_t + \delta.
\] (6)

The demand curve facing each local monopolist is again given by the marginal product schedule:

\[
p_{it} = \frac{\partial Y_t}{\partial x_{it}} = \alpha A_{it}x_{it}^{\alpha-1}
\] (7)

So her profit-maximization problem is now:

\[
\pi_{it} = \max_{x_{it}} \{ p_{it}x_{it} - R_tK_{it} \} = \max_{x_{it}} \{ \alpha A_{it}x_{it}^{\alpha} - R_tA_{it}x_{it} \}
\]

It is immediate to see that the solution \( x_{it} \) to this program is independent of \( i \), that is, in equilibrium all intermediate firms will supply the same quantity of intermediate product.
Thus, for all $i$:
\[
\frac{K_{it}}{A_{it}} \equiv x_t \equiv \frac{K_t}{A_t} = m_t. \tag{8}
\]

where $K_t = \int K_{it} \, di$ is the aggregate demand for capital which in equilibrium is equal to the aggregate supply of capital; and $A_t = \int A_{it} \, di$ is the average productivity parameter across all sectors. The average $A_t$ can be interpreted as the number of efficiency units per worker, because substituting (8) into (4) yields:
\[
Y_t = (A_t L)^{1-\alpha} K_t^{\alpha}. \tag{9}
\]

Therefore $m_t = K_t/A_t$ is the capital stock per effective worker.

The first-order condition for the above maximization problem is simply:
\[
\alpha^2 A_{it} x_{it}^{\alpha-1} - A_{it} R_t = 0. \tag{10}
\]

Using this condition to substitute away for $R_t$ in the monopolist’s profit, together with (8), we immediately get:
\[
\pi_{it} = A_{it} \alpha (1 - \alpha) m_t^{\alpha}. \tag{11}
\]

### 4.1.3 Innovation and research

As in Chapters 4 and 5, an innovation in sector $i$ at time $t$ increases productivity in that sector from $A_{it}$ to $\gamma A_{it}$, where $\gamma > 1$. Also, as in the continuous time model of chapter 4, innovations in each sector arrive at a rate proportional to the resources devoted to R&D in that sector:
\[
\lambda n_{it}.
\]

The input $n_{it}$ is R&D expenditure $N_{it}$ (of the final good) divided by the current productivity parameter:
\[
n_{it} = N_{it}/A_{it}
\]

which again incorporates the “fishing-out” effect whereby innovating becomes more difficult as technology advances. As we shall see, productivity-adjusted R&D will be the same in all sectors:
\[
n_{it} = n_t \text{ for all } i
\]
4.1.4 The research arbitrage equation

Let $V_{it}$ denote the value of an innovation at time $t$ to an innovator in sector $i$. The flow of profit accruing to a researcher in sector $i$ that invests $N_{it}$ units of final good in R&D at time $t$, is:

$$\lambda \frac{N_{it}}{A_{it}} V_{it} - N_{it}$$

where the first term is the probability of an innovation per unit of time $\lambda n_t$ multiplied by the value of the innovation, and the second term is the flow of research expenditure. Free entry into research implies that this expected profit flow must equal zero, so that:

$$\lambda v_t - 1 = 0 \quad (12)$$

where $v_t = V_{it}/A_{it}$ is the productivity-adjusted value of an innovation at $t$.

Suppose that each monopolist gets to keep her profit until she is replaced by the next innovation. Then $v_t$ is the expected present value of the productivity-adjusted stream of profit $\bar{\pi}_\tau = \pi_{ir}/A_{it}$ over all future dates $\tau$ from $t$ until the next innovation occurs, given the arrival rate $\lambda n_\tau$ and the rate of interest $r_\tau$ at each future date $\tau$. The post-innovation productivity $A_{i\tau}$ in sector $i$ will stay constant, and equal to $\gamma A_{it}$, until the next innovation, so we have:

$$\bar{\pi}_\tau = \pi_{ir}/A_{it} \equiv \gamma \alpha (1 - \alpha) m_t^\alpha. \quad (13)$$

As in the continuous time model of Chapter 4, given the assumption of risk neutrality, $v_t$ must satisfy the asset equation:

$$r_t v_t = \bar{\pi}_t - \lambda n_t v_t.$$  

The left-hand side is the income you could get from selling the stream for the price $v_t$ and investing it at the rate of interest $r_t$. The right-hand side is the expected return from retaining ownership of the stream - the flow of profit $\bar{\pi}_t$ minus a capital loss of the entire value $v_t$ with probability $\lambda n_t$ (the probability per unit of time that an innovation will render the product obsolete).\(^6\)

Rewriting this asset equation we get the same formula:

$$v_t = \frac{\bar{\pi}_t}{r_t + \lambda n_t}.$$  

\(^6\)In general, the return to an asset ought to include a continuous capital-gain term $\dot{v}_t$ that will accrue if no innovation occurs. But in this case the free-entry condition guarantees that $v_t$ is a constant, so that $\dot{v}_t = 0$. 

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as we saw in the continuous time model of chapter 4. That is, the productivity-adjusted value of an innovation is the productivity-adjusted profit flow discounted at the obsolescence-adjusted interest rate \( r_t + \lambda n_t \). Substituting this formula into the free-entry condition (12) and using the relationship (13) gives us immediately:

\[
1 = \lambda \frac{\gamma \alpha (1 - \alpha) m_t^\alpha}{r_t + \lambda n_t}.
\]  

(RA)

In addition to the usual comparative statics results of the Schumpeterian model, this research arbitrage equation implies the R&D input \( n_t \) will be an increasing function of the capital stock per efficiency unit of labor, \( m_t = K_t/A_t \). This is because of a scale effect, according to which more capital per person means more demand per intermediate product, which implies a bigger reward to innovation and hence a larger equilibrium rate of innovation.

### 4.1.5 Productivity growth

The expected growth rate of each productivity parameter \( A_{it} \) is just the flow probability of an innovation \( \lambda n_t \) times the size of innovation \( \gamma - 1 \). Since \( A_t \) is just the average of all the \( A_{it} \)'s then its expected growth rate \( g_t \) will be the same as each component of the average. By the law of large numbers this will be not just the expected growth rate of \( A_t \) but also the actual growth rate of \( A_t \). That is:

\[
g_t = \dot{A_t}/A_t = (\gamma - 1) \lambda n_t
\]

Together with the research arbitrage equation this makes the growth rate an increasing function of the capital stock per efficiency unit and the rate of interest:

\[
g_t = (\gamma - 1) \left( \lambda \gamma \alpha (1 - \alpha) m_t^\alpha - r_t \right)
\]

But the rate of interest is just the rental rate (6) minus the depreciation rate \( \delta \), and by (10) the rental rate

\[
R_t = \alpha^2 x_t^{\alpha - 1} = \alpha^2 m_t^\alpha - 1
\]

is a function of \( m_t \). So we can express the growth rate as:

\[
g_t = g(m_t).
\]  

(14)

According to (14), the labor-augmenting productivity growth rate is an increasing function.

\[\text{Specifically, } g(m_t) = (\gamma - 1) \left( \lambda \gamma \alpha (1 - \alpha) m_t^\alpha - \alpha^2 m_t^{\alpha - 1} + \delta \right).\]
tion of the capital stock per efficiency units. This is because when \( m_t \) rises it stimulates innovation, through two channels. The first channel is the scale effect described in the previous section, through which more capital per person means more demand per sector for improved intermediate products. The second channel is an interest-rate channel; more capital means a smaller equilibrium rental rate and hence a smaller equilibrium rate of interest, which stimulates innovation by reducing the rate at which the expected profits resulting from innovation are discounted.

### 4.1.6 Complete model and its implication for growth accounting

Two of the equations of this model, namely the capital accumulation equation (5) and the aggregate production function (9), are identical to the two equations on which the neoclassical model of Solow and Swan was built. It is straightforward to derive from these two equations the fundamental differential equation of the neoclassical model:

\[
\frac{dm}{dt} = sm^\alpha - (\delta + g) m
\]  

(15)

In this case we are dealing with the case in which population is constant but the productivity parameter \( A_t \) is growing at the rate \( g \), so (15) is the same as the corresponding equation in chapter 1.

But instead of taking the rate of technological progress \( g \) as given, the present model has endogenized it. Specifically, \( g \) depends on the capital stock per efficiency unit \( m \) according to:

\[
g = g(m) \quad \text{(16)}
\]

Equations (15) and (16) together produce a hybrid model that incorporates and generalizes both the neoclassical model and the Schumpeterian model, as shown in Figure 1 below. This curve shows how the model converges to a unique stationary state \((m^*, g^*)\) in which both the capital stock per efficiency unit and the rate of technological progress are constant over time.

**FIGURE 1 - FROM FIGURE 3.3 OF FIRST EDITION**

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Since \( m = K/A \), therefore:

\[
\frac{1}{m} \frac{dm}{dt} = \frac{1}{K} \frac{dK}{dt} - \frac{1}{A} \frac{dA}{dt} = sA^{1-a}K^\alpha - \delta - g = sm^{\alpha-1} - \delta - g
\]

Multiplying the first and last expressions in this string of equalities by \( m \) yields (15).
The steady state is characterized by two equations; namely the growth equation (16) which shows how $g$ depends positively on $k$ and the neoclassical steady-state equation:

$$m = \left( \frac{s}{\delta + g} \right)^{\frac{1}{1-\alpha}}$$

which shows how $m$ depends negatively on $g$. These two equations are represented respectively by the curves $RR$ and $KK$ in Figure 2 below.

**FIGURE 2 HERE**

An interesting implication of this analysis is that by endogenizing the rate of technological progress in the neoclassical model we allow the long run growth rate to depend not just on the incentives to perform R&D but also on the saving rate. An increase in the productivity of R&D $\lambda$ or the size of innovations $\gamma$ will shift the $RR$ curve up, resulting in an increase in steady-state growth, whereas an increase in the saving rate $s$ will shift the $KK$ curve to the right, resulting in an increase in $k$ and also an increase in the steady state growth rate.

### 4.2 Implications for growth accounting

Because it endogenizes productivity growth as well as capital accumulation, this hybrid model provides a more complete interpretation of the results of growth accounting than does the neoclassical model which takes productivity growth as exogenously given.

Like the neoclassical model, the hybrid model implies that in the long run the capital stock per efficiency unit $m = K/AL$ will be constant, so that in the long run the growth of capital per person:

$$k = K/L = Am$$

will equal the rate of technological progress:

$$\dot{k}/k = g.$$

But this does not mean that capital deepening ($\dot{k}/k$) is *caused* by $g$, just that they are equal.

Indeed once we endogenize the rate of technological progress we cannot meaningfully speak of it as causing anything. An analogy from supply and demand theory might help. That theory implies that the quantity supplied must equal the quantity demanded. But this does not mean that supply causes demand, or that demand causes supply, just that both are endogenously determined, in the same way, by those factors that underlie the demand and supply schedules. The one exception is when we take supply as given inelastically, in which
case a change in supply (shift in the supply curve) is the only thing that can cause a change in the quantity demanded.

Likewise in growth theory, once we go beyond the simple theory in which we take $g$ as given exogenously we can only say that both capital deepening and productivity growth are endogenously determined by the factors underlying the two curves of Figure 2. So for example, when a change in the incentives to perform R&D change, this will result in a higher $g$ which we can meaningfully attribute to the force of innovation, since it was the innovation side of the economy that was altered. In this case the hybrid model agrees with the Solow model. But when the saving rate $s$ changes this will displace the KK curve in Figure 2 to the right, again causing both $g$ and $\dot{k}/k$ to go up, and in this case both changes are attributable to capital accumulation, since it was a change in thrift not a change in innovation that caused the shift.

In both of these cases a growth accountant will ultimately conclude that the fraction $\alpha$ of the change in growth was accounted for by capital deepening and $1 - \alpha$ by TFP growth. We know this because this is the implication of the Cobb-Douglas production function (9), as we explained in section 3.2 above. Yet in one case it was all caused by innovation and in the other case it was all caused by capital accumulation.

As these examples illustrate, to estimate the extent to which growth is caused by either of these two forces we need to identify the causal factors that shift the two curves, estimate by how much they shift the curve, estimate the slopes of the curves and then measure the amount by which the causal factors have changed over the time period in question. One of the main objects of the rest of this book will be to identify the causal factors that shift the curves, and to survey some of the evidence that would allow us say to which of these factors have been primarily responsible for economic growth at different times and in different countries.